Firms' Heterogeneity and Aggregate Fluctuations: What Can We Learn From Machine Learning?

Marco Errico
Bank of Italy

Simone Pesce
Boston College

Luigi Pollio*
Boston College

Preliminary. Comments are welcome.

First Draft: June 27, 2024

Abstract

The heterogeneous sensitivity of firms to aggregate fluctuations influences the dynamics of the business cycle. We study how firms' outcomes (sales, debt, investment, and market value) respond to aggregate fluctuations (business cycle, monetary policy, uncertainty, and oil shocks) based on eight observed characteristics using the Generalized Random Forest algorithm. Using micro-level data from Compustat, we document three micro-facts about the cross-sectional heterogeneity in firm sensitivity: (1) while the linear OLS benchmark provides good estimates of the average effect, there is significant cross-sectional heterogeneity in firms' sensitivities; (2) the importance of firm characteristics varies across dependent variables and shocks, but on average, non-financial characteristics play a larger role in explaining the heterogeneity in sensitivity; (3) sensitivity to aggregate fluctuations exhibits non-linear patterns with respect to firms' characteristics. We study the implications of cross-sectional heterogeneity in firm-level sensitivity, developing an aggregation theory and leveraging the estimated random forest model to generate counterfactual firm-level sensitivities. We show that: (1) the heterogeneity in sensitivity amplifies the aggregate response to macroeconomic variables; (2) the impact of estimated non-linearity at the micro-level on the aggregate response is negligible; (3) non-financial characteristics shape aggregate responses more than financial characteristics; and (4) state-dependent heterogeneity in firm characteristics plays a marginal role in driving the aggregate response to macroeconomic variables.

JEL Codes: D22, E32, C14, E5
Keywords: Firm Heterogeneity, Firm Sensitivity, Machine Learning, Aggregate Fluctuations, Monetary Policy, Business Cycle, Uncertainty, Oil Shock.

*Contact: Luigi Pollio, lpollio1@umbc.edu; Marco Errico, marco.errico@esterni.bancaditalia.it; Simone Pesce, simone.pesce@bc.edu. Part of the research was conducting while Simone Pesce was visiting the European Central Bank. The views expressed here do not necessarily reflect those of the Bank of Italy or the Euro-System.

1 Introduction

Cross-sectional heterogeneity in firm sensitivity to aggregate fluctuations is a crucial area of study in macroeconomics and finance, for its implications on aggregate outcome and policy conduct. It examines how firms respond to fluctuations in key macroeconomic variables such as GDP growth and interest rates, and seeks to understand the implications of these fluctuations for firm performance, investment decisions, and risk management strategies (Ottonello and Winberry, 2018; Crouzet and Mehrotra, 2020; Alfaro et al., 2024). By studying the cross-sectional heterogeneity in firm sensitivity to aggregate fluctuations and its underlying drivers, researchers gain insights into the dynamics of business cycles, the transmission of shocks across the economy, and the factors driving variations in firm-level and aggregate outcomes during different phases of the economic cycle. Since observable characteristics deeply influence firms' exposure to aggregate fluctuations, understanding the role of firms' heterogeneity along multiple dimensions is essential for evaluating the amplitude of macroeconomic shocks and the effectiveness of policy interventions.

In this paper, we study how firms' outcomes respond to aggregate fluctuations based on observed characteristics using a machine learning approach. We employ the Generalized Random Forest (GRF), a newly developed causal model introduced by Athey et al. (2019), to estimate firms' responses to aggregate fluctuations. Most of the macroeconomic literature estimates micro-level sensitivity to aggregate fluctuations using ordinary least squares (OLS). However, OLS has several statistical limitations, such as the curse of dimensionality and its inability to account for non-linear relationships between the variable of interest and firms' characteristics. In contrast, machine learning algorithms allow for the analysis of firms' responses based on a large set of balance sheet characteristics, which would be impractical with a standard linear regression model. Additionally, these algorithms do not impose a specific functional form on the relationships between outcomes and characteristics, allowing for greater flexibility and richer specifications. Our analysis aims to build on previous findings in the literature and uncover new empirical evidence on the heterogeneity of firms' responses to aggregate fluctuations and its aggregate implications.

We begin by documenting a set of micro-facts about firms' heterogeneity and their responses to aggregate fluctuations based on the estimated random forest model. We utilize U.S. firm-level data from Compustat, spanning the years 1990 to 2018, and focus on key firm-level outcome variables extensively studied in the literature, such as sales, investment rate, debt issuance, and market value. Our study investigates how joint cross-sectional heterogeneity across eight distinct observed characteristics affects firms' sensitivity to business cycle fluctuations and to three specific types of exogenous shocks: monetary policy shocks, uncertainty shocks, and oil price shocks. The characteristics we consider, which have been individually examined in prior research, include leverage ratio, liquidity, distance-to-default, proportion of short-term debt, size, return on assets, sales volatility,

and industry scope.

First, we document and confirm the presence of significantly large heterogeneity in firms' sensitivities to business cycle fluctuations, even though the linear benchmark (OLS) provides good estimates of the average effect. This suggests that although the average estimates tend to be unbiased by firms' heterogeneity, machine learning approaches, such as the GRF, can estimate substantial variations in firms' sensitivities. This includes instances where firms exhibit opposite sign responses to the same aggregate shock, such as investment and debt rates in response to monetary policy.

Second, we investigate how firms' responses to aggregate fluctuations vary by firm characteristics and over time. One of the key advantages of machine learning approaches is their ability to relax the curse of dimensionality and consider a large set of variables. In our specific case, we quantify the role that each of the ten firm characteristics plays in shaping the cross-sectional heterogeneity in firm sensitivity. On average, a firm's size and distance-to-default account for the most significant heterogeneity in sensitivity. Importantly, the relevance of these characteristics varies across dependent variables and shocks: financial characteristics are particularly relevant for business cycle and uncertainty shocks, while non-financial characteristics are more pertinent under monetary policy shocks.

Third, we uncover non-linear relationships between firms' sensitivities and characteristics, highlighting the limitation of the OLS specification. The standard OLS approach entails the use of an interaction term between the aggregate shock and firm characteristics to assess the cross-sectional differences in firm sensitivity along a specific characteristics. The estimated random forest allows us to fully capture in a non-parametric fashion the relationship between firms' characteristics and sensitivity. We show the presence of several (inverted) U-shaped or kinked patterns in the relationship between firms' characteristics and sensitivity across all scenarios considered, such as the absence of any marginal effect of monetary policy shock for firms with medium-high distance-to-default.

Motivated by these micro-evidences, we study the implications of cross-sectional heterogeneity in firm-level sensitivity for the aggregate economy. We develop an aggregation theory that allows us to connect the individual, firm-level response to the aggregate outcome and leverage on the estimated random forest model to generate counterfactual firm-level sensitivities. Specifically, we focus on three different counterfactual exercises: we assume that all firms have the same sensitivity in order to assess the impact of cross-sectional heterogeneity on aggregate responses; we assume that all firms have the same underlying characteristics (or subset of) to evaluate their impact and the role of non-linearities on aggregate responses; lastly, we investigate whether endogenous variations in firms' characteristics influence the response to aggregate shocks during periods of recession.

We show that abstracting away from the heterogeneity in sensitivity amplifies the aggregate response to macroeconomic variables, driven primarily by the low sensitivity of large firms in the

benchmark scenario. Additionally, we find that the role that non-linearity has for the aggregate response is negligible, suggesting that firm characteristics are not heavily distributed on areas with strong non-linearities. We instigate the relative aggregate importance of financial and non-financial characteristics, and show that the distribution of non-financial characteristics crucially shapes aggregate responses, consistent with our micro-level findings. Finally, fixing the distribution of sensitivities at pre-recession levels does not result in significant changes in aggregate responses, except for the widely-studied case of investment and monetary policy, suggesting that state-dependent heterogeneity plays a marginal role in driving the aggregate response to macroeconomic variables.

The remainder of the paper is organized as follows. Section 2 provides information on the data and empirical methodologies, OLS and random forest based on GRF. Section 3 presents the three empirical facts on micro-level sensitivities. Section 4 proposes an aggregation theory, describes the counterfactual exercises, and the results on the aggregate implications. Section 5 concludes.

Literature. This paper is related to several strands of literature. First, it contributes to the recent and rapidly growing field applying machine learning techniques, such as random forests, to economic analysis (Athey et al., 2019). Few works have applied these techniques to study firm-level heterogeneous sensitivity and macroeconomics in general.[1] The closet to our work is Paranhos (2024), which examines the relationship between firms' default risk and the effectiveness of monetary policy transmission. By generalizing standard local projection methods non-parametrically, she reveals non-linearities in the effect of monetary policy shocks conditional on firm risk levels. Our contribution extends these works by applying the random forest models to comprehensively study firms' heterogeneity in sensitivity to aggregate fluctuations, incorporating all available information related to firms' balance sheets and various macroeconomic shocks highly relevant in the literature and offering a holistic analysis of the factors driving variations in firm-level outcomes.

Second, it contributes to the literature that studies the heterogeneity of firms' sensitivity to aggregate fluctuations depending on balance sheet characteristics. The literature has investigated the excess sensitivity to monetary policy by looking at different metrics of the firms' performance and proxies for financial frictions. A non-exhaustive list of important contributions includes Ottonello and Winberry (2018) study the role of leverage in the transmission of monetary policy with respect to investment using Compustat data; Jeenas (2018a) shows that cash holdings may be a better proxy for monetary policy heterogeneity.[2] Similarly, Crouzet and Mehrotra (2020) and Begenau

[1] On the consumers side, Khazra (2021) explores the heterogeneity of house price elasticity of consumption using micro panel data via the GRF model, finding that neglecting local heterogeneities in elasticity leads to overestimating the total consumption response during housing market booms and busts.

[2] Other papers in the literature study more generally the implication of heterogeneity in firm or industry behavior in response to monetary policy shocks Gaiotti and Generale (2002), Ehrmann and Fratzscher (2004), Peersman and Smets (2005), Cloyne et al. (2018).

and Salomao (2019) focus on the role of size in the response to business cycle fluctuations; Alfaro et al. (2024) shows that real and financial frictions amplify the negative impact of uncertainty shocks.[3] Despite the recent importance of energy price shocks, heterogeneity in the response to oil shocks is relatively underexplored.[4] Previous work usually considers one balance sheet characteristics at the time. Departing from the standard linear OLS strategy and using machine learning allow our study to complement and expand this literature by providing a comprehensive analysis across the set of characteristics considered.

Third, our paper relates to the literature examining the implications of micro-level empirical heterogeneity for macroeconomic aggregates. We develop an aggregation theory in the spirit of Crouzet and Mehrotra (2020), and leverage the distribution of estimated firm-level sensitivities from the random forest model to decompose the aggregate effect into a mean and a covariance term. This extends the analysis in Crouzet and Mehrotra (2020), which focuses on two arbitrary groups (small vs large firms), without taking into account the whole cross-sectional distribution. An alternative approach is denoted by Chang et al. (2024a), which consists on a functional VAR that integrates aggregate variables with cross-sectional distributions to study their dynamic interactions. They find that including the earnings distribution in a standard VAR has little effect on the dynamics of aggregate variables following aggregate shocks. In another study, Chang et al. (2024b) demonstrate that conventional monetary policy shocks reduce earnings inequality, using cross-sectional distributions of U.S. earnings, consumption, and financial income. In contrast to these works, we utilize machine learning methods to estimate micro sensitivities to aggregate shocks, which we then use to study aggregate implications via a bottom-up approach. This methodology allows us to capture the nuanced impacts of firm-level heterogeneity on macroeconomic outcomes.

2 Empirical Analysis

2.1 Data and measurement

Our primary source of data is the quarterly Compustat dataset, which provides comprehensive financial statement information for publicly listed companies in the U.S. We merge firm-level data with a set of aggregate variables and shocks commonly used in the literature. Our final dataset includes 220,259 firms with quarterly financial information spanning from 1990-Q1 to 2018-Q4. The start and end dates align with the dates of the aggregate variables in the panel, excluding the period of Covid-19. Additionally, required variables are deflated using the implied price index of

[3] Kumar et al. (2023) shows that the effects of uncertainty on sales and investment are heterogeneous depending on firm size using an RTC design.

[4] A few works show that the response of firm's market value to oil shocks depends on its size and industry scope (Narayan and Sharma, 2011; Tsai, 2015).

gross value added in the U.S. non-farm business sector. For further details on variable construction and data cleaning processes, please refer to Appendix A. Below, we provide a brief overview of the primary firm-level variables and the measurement of aggregate variables.

Firm-level data. Our empirical analysis utilizes two sets of firm-level variables. The first set includes four outcome variables to study their heterogeneous responses: annual real sales growth, debt issuance (measured by the 1-year percentage change in short and long term debt), market value growth, and the investment rate (measured as the 1-year percentage change in capital stock using the perpetual inventory method). The second set consists of eight explanatory variables representing general firm balance sheet characteristics, which we divide into two groups, financial and non-financial variables. Non-financial variables include firm' size (measured by the logarithm of total assets), industry scope (captured by NAICS 5-digit industry codes), 10-years sales volatility, and return on assets (ROA) to capture firm profitability. Financial variables include liquidity ratio (cash-to-total assets), leverage ratio (total debt-to-total assets), distance to default to measure the probability of default (Merton, 1974), and the proportion of short-term debt to total debt to measure debt liquidity. These firm balance sheet characteristics have been extensively used in the literature to study the transmission of aggregate fluctuations onto firms' choices.[5] Appendix A presents selected summary statistics and histograms for the firm-level variables used in the empirical analysis. Importantly, Table 4 in Appendix A reports the pairwise correlation between all independent variables, showing that, although some correlation exists among firm characteristics, they provide different information along different margins.

Aggregate fluctuations. We investigate the sensitivity of firms' observables to the following aggregate fluctuations: business cycles, macro uncertainty shocks, monetary policy shocks, and oil price shocks[6]. Business cycle fluctuations are proxied by the percentage annual change in real GDP following Crouzet and Mehrotra (2020). Monetary policy shocks are measured using interest rate surprises around Fed announcements and are cleaned of past aggregate fluctuations (Bauer and Swanson, 2023). Uncertainty shocks are exogenous change in macroeconomic uncertainty as measured in Jurado et al. (2015). Oil price shocks are proxied with high-frequency changes in oil

[5]For instance, Ottonello and Winberry (2018), Cloyne et al. (2018), and Jeenas (2018a) study the role that distance to default, leverage and cash play in the transmission of monetary policy shocks to investment, respectively. Similarly, Alfaro et al. (2024) studies the effects of uncertainty on firms' financial variables such as cash and leverage, while Crouzet and Mehrotra (2020) focuses on how size and industry impact the response to business cycle fluctuations.

[6]We examine oil price shocks for two reasons. First, they provide a clear and distinct example of exogenous inflation changes driven by supply factors. Second, oil price shocks have gained increasing importance in the macroeconomic literature, particularly following the Covid-19 pandemic.

supply expectations around OPEC announcements from Känzig (2021).[7] To normalize the size of the shocks, we use them as instruments for a set of endogenous variables (Stock and Watson, 2018).[8] Specifically, we use the one-year percentage change in the 1-year government bond yield for monetary policy shocks, the one-year change in the oil price index for oil price shocks, and the volatility index for uncertainty shocks. We perform the analysis separately for each aggregate variable and combine the results for interpretation. Figure 11 in Appendix A presents the time series of the aggregate variations used in the paper.

2.2 Methodology

In this section, we outline the key features of the two empirical methodologies used to estimate the heterogeneous response of firms to aggregate shocks, a benchmark linear regression (or its dynamic version, Local Projection (Jordà, 2005)) and a Generalized Random Forest algorithm based on Athey et al. (2019). We also discuss advantages and disadvantages of the two methodologies.

Linear Model. Linear regression models are heavily used to estimate the heterogeneous effects of aggregate fluctuations on firms' observables. Let the index i as the firm index and t as the time period, the econometrician can estimate the heterogeneous effects of aggregate fluctuations using the following linear model via Ordinary Least Squares (OLS):

$$Y_{i,t} = \alpha + \beta_0 X_{i,t-1} + \beta_1 W_t + \beta_2 X_{i,t-1} W_t + \varepsilon_{i,t}, \tag{1}$$

where $Y_{i,t}$ represents the dependent variable of interest, $X_{i,t-1}$ is a finite vector of predetermined independent variables that account for specific firm-level characteristics, and W_t is a variable capturing aggregate fluctuations[9]. The coefficient of interest is the one on the interaction term between the vector of characteristics, $X_{i,t-1}$, and the aggregate fluctuation, W_t (β_2), which captures the heterogeneous effect of variations in W_t on firms' observable $Y_{i,t}$ due to firms' heterogeneity along the characteristics $X_{i,t-1}$. This suggests how individual characteristics influence firms' responses in the variable $Y_{i,t}$ to aggregate fluctuations.

Two limitations arise from estimating the heterogeneous response to aggregate fluctuations as in Equation (1). First, the model in Equation (1) assumes a proportional relationship in the

[7] Monetary policy shocks are identified using high-frequency variations in the 3-month federal funds rate futures. The shock series are available on the website of the paper. For more details, please refer to Bauer and Swanson (2023). Similarly, Känzig (2021) isolates exogenous variation in oil price by looking at how oil futures prices change around OPEC announcements.

[8] Using the exogenous variables as instrument imposes a unit effect normalization of the shocks in terms of a 1-unit change in the endogenous variable (Stock and Watson, 2018).

[9] Fixed effects that allow capturing time-invariant differences in firm observable behavior across firms are omitted in the specification.

heterogeneous average effect due to its linearity. For example,when comparing two firms where $X_{i,t-1}$ is 10% for one firm and 90%, for the other, the model predicts that the effect of a change in W_t on Y_t for the second firm will be nine times that of the first firm. This proportional relationship is often inconsistent with theoretical expectations. Second, the linear model struggles to account for heterogeneity when there are many covariates, as it can become overparameterized and lose interpretability. These limitations restrict the model's ability to provide detailed insights into how different factors influence the heterogeneous response, potentially overlooking complex interactions and non-linear relationships common at the micro level in response to aggregate fluctuations.

Generalized Random Forest. Machine learning offers an alternative method to estimate the heterogeneous effects to aggregate fluctuations. We employ the method developed by Athey et al. (2019) to study the heterogeneous effect of aggregate fluctuations on firms' observables. Formally, we estimate the following random forest model:

$$Y_{i,t} = \beta\left(X_{i,t-1}\right) W_t + \varepsilon_{i,t}\ , \quad \beta\left(x\right) = \mathbb{E}\left[\beta(X_{i,t-1}) | X_{i,t-1} = x\right]. \tag{2}$$

$\beta(x)$ is estimated non-parametrically via generalized random forest:

$$\hat{\beta}(x) = \frac{\sum_{i=1}^{n} \alpha_i(x)\left(W_i - \bar{W}_\alpha\right)\left(Y_i - \bar{Y}_\alpha\right)}{\sum_{i=1}^{n} \alpha_i(x)\left(W_i - \bar{W}_\alpha\right)}, \tag{3}$$

where α_i is a weight determined by the causal forests , $\bar{W}_\alpha = \sum \alpha_i(x) W_i$ is a weighted average treatment of shock W_i, and $\bar{Y}_\alpha = \sum \alpha_i(x) Y_i$ is a weighted average outcome for variable Y_i.[10]

Compared to the OLS estimator in Equation (1), the parameter of interest $\beta(x)$ captures the average effect of shock W_i for firms with similar characteristics. In a nutshell, the algorithm undertakes the generation of several trees, in which the sample is partitioned into subsamples (bootstrapped) based on predictors X_i, and for each tree the weight $\alpha_i(x)$ reflects the frequency with which the observation i ends up in the same "leaf" as x. All the causal forest are run with clusters at firm level. We leave more in-depth details on the algorithm to Athey et al. (2019). Appendix A provides more details and discussion of the algorithm used in the paper.

Advantages of GRF. GRF offers two key advantages over OLS for estimating heterogeneous treatment responses. Firstly, GRF is immune from the curse of dimensionality in datasets with a large number of covariates, where traditional statistical methods like OLS often struggle due to issues like multicollinearity and overfitting. In our case, if we wanted to include all possible interactions among the vector of eight characteristics we consider would require a linear model

[10]Furthermore, because we consider year-to-year variations as the dependent variable, we lag the vector of balance sheet characteristics by four periods.

with 256 parameters, clearly overparameterized and difficult to interpret. Strictly related to this point, the GRF is not subject to the arbitrary choices of the econometrician in terms of model specifications and set of firms' characteristics to include, which make comparisons across models difficult and unprecise. Secondly, GRF allows for non-linearities between the outcome variable and characteristics, as well as among explanatory characteristics, by employing an ensemble of decision trees to estimate treatment effects for different subgroups within the data. Consequently, we can more effectively identify and quantify the heterogeneity in treatment responses, which is particularly suitable in our case when studying how the impact of aggregate fluctuations vary across firms.[11] In particular, the GRF estimates individual-level coefficients, in contrast to OLS: in other words, GRF allows the econometrician to observe and study the entire distribution in estimates conditional on the covariates.

3 A Set of Micro Facts

We document three key micro-facts about firms' varying sensitivity to aggregate fluctuations. First, while the average effect estimated via causal forest is similar to linear model estimates, it conceals significant heterogeneity at the firm level. Second, unlike current literature, we find that non-financial characteristics account for the majority of firms' sensitivity to aggregate fluctuations. Lastly, we show that most firm-level characteristics influence sensitivity in a non-linear manner.

Fact 1: The average effect estimated via casual forest, while close to the estimates of a linear model, summarizes highly heterogeneous sensitivities at the firm level.

We compare the outcome of the causal forest with the benchmark average effect estimated using a linear regression. Figure 1 shows the distribution of the firm-level sensitivities to aggregate fluctuations for each aggregate shock-outcome variable pair from the causal forest in Equation (2), and compares them with the average sensitivity estimated using OLS regression[12]. We can draw two major conclusions from Figure 1.

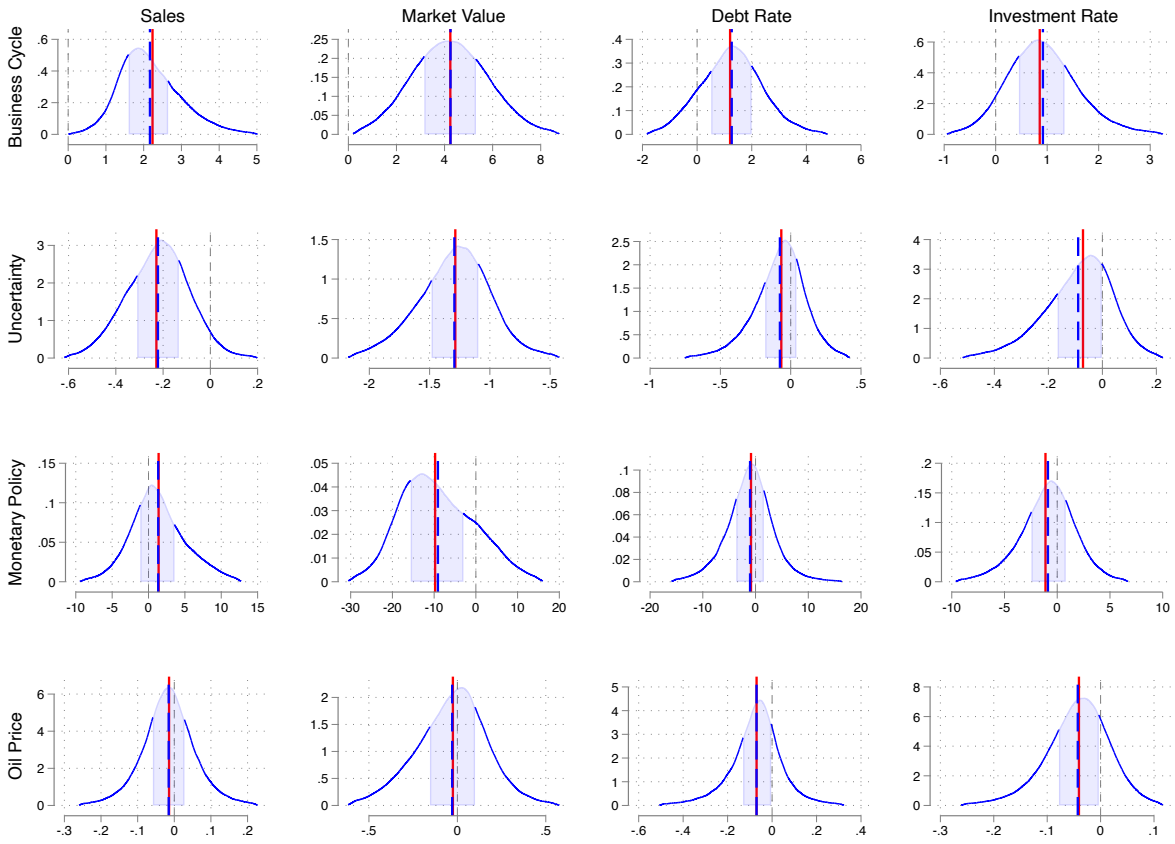
Similarity in the average effect First, the average effects of aggregate shocks estimated using the causal forest (blue dashed line) are very close to the average estimates from a standard linear model (red solid line).[13] The signs of the shocks on firms' level observables align with

[11] See Section 2.1 and Appendix A for additional details on data construction, econometric techniques, and relation to previous specifications.

[12] The estimated firm-level sensitivities are standardized to reflect a 1 percent increase in an aggregate variable. As an example, a value of 2 in the firm's sensitivity means that an aggregate shock that increase the endogenous variable by 1 percent is associated with a firm's outcome variable increase by 2 percent.

[13] We discuss in Appendix B how we construct the average effect from the causal forest model.

Figure 1: Comparison outcome causal forest and linear model



Notes: The figure shows the distribution of micro-level sensitivities to aggregate fluctuations estimated using the causal forest. Each cell represents an outcome-shock pair, with shocks on the rows and outcome variables on the columns. The business cycle is measured as the year-to-year growth rate in real GDP per capita. Monetary policy is the monetary policy shock from Bauer and Swanson (2023) and normalized to change the 1-year government yield by 1 percent in a year. Uncertainty is the 12-months macro uncertainty shock from Jurado et al. (2015) and normalized to change the volatility index by 1 percent in a year. Oil price is the oil price shock from Känzig (2021) and normalized to increase the oil price by 1 percent in a year. The dashed blue line indicates the average estimate, while the shaded blue area represents the interquartile range of micro-sensitivities from the causal forest. The solid red line indicates the average effect of a shock on the outcome variable based on the linear benchmark model, accounting for unobserved heterogeneity at the NAICS-5 digit industry level. Firms' sensitivities are trimmed at 0.5% on both sides.

economic intuition, and the magnitudes of the average effects are also consistent with previous literature. For instance, GRF estimates that, on average, a 1 percent increase in GDP is associated with an increase in the average firm sales by 2.23 percent, which is close to the 3 percent found by Crouzet and Mehrotra (2020).[14] An exogenous increase in the VIX by 1 percent due to a spike in macro uncertainty negatively affects firms' outcomes, reducing average firm sales by 0.20 percent, market value by 1.28 percent, debt rate by 0.06 percent, and investment rate by 0.09 percent, in line with the results in Alfaro et al. (2024). Unexpected interest rate hikes due to monetary policy interventions have a profound negative effect on average firm market returns, debt issuances, and investments in tangible assets. Finally, an unexpected spike in the oil price consistently depress sales and market value and have a slightly negative effect on debt issuances and capital investment.[15]

Heterogeneity in firms' sensitivity Second, while both the causal forest and linear models estimate similar average effects, the causal forest model allows us to unveil significant heterogeneity in firms' sensitivity to aggregate fluctuations. Figure 1 shows that the micro-level sensitivities derived from the causal forest are very dispersed around the average effect. The IQR (shaded blue area) is close around the means, indicating long tails and suggesting that firms potentially exhibit significantly different sensitivities. Similarly, Table 13 in Appendix C reports the median coefficient of variation (CV) over time for each aggregate shock-outcome pair, confirming that more than 80% of the cases we examined exhibit either moderate (CV $\in [0.5, 1]$) or high (CV > 1) levels of dispersion in firm-level predicted sensitivities. We confirm the same qualitative conclusions using the machine-learning based test for heterogeneity in treatment effects proposed by Chernozhukov et al. (2018).[16]

Importantly, the heterogeneity in firms' sensitivity does not necessarily imply that the sensitivity exhibits the same sign for all firms. For instance, outcome variables such as investment and debt rate always exhibit tails in the distribution of sensitivities with signs opposite to the sign of the average effect. This result highlights the relevance of using tools like causal forest that allow for the estimation of the rich heterogeneity in firms' sensitivity to aggregate shocks. This has potential implications for the aggregate response and for policy conduct, as we explore in Section 4.

[14] Crouzet and Mehrotra (2020) finds that a 1 percent increase in GDP is associated with a 3% increase in the average firm sales using QFR firm-level data.

[15] For a more immediate comparison, Table 5 in Appendix C reports the estimated average effect for each aggregate shock-outcome variable pair from the causal forest in Equation (2) and the linear model in Equation (1). They also include the t-statistics from a standard t-test to determine if there is a significant difference between the estimated average effects from the causal forest and the linear model. In none of the cases analyzed do we find a statistically significant difference between the machine learning and linear models.

[16] Appendix B provides details on the construction of the test. Table 13 in Appendix C reports the coefficients and the corresponding p-values of the heterogeneity in treatment effects test.

Fact 2: On average, firm's size and distance-to-default play a predominant role in explaining the heterogeneity in sensitivity.

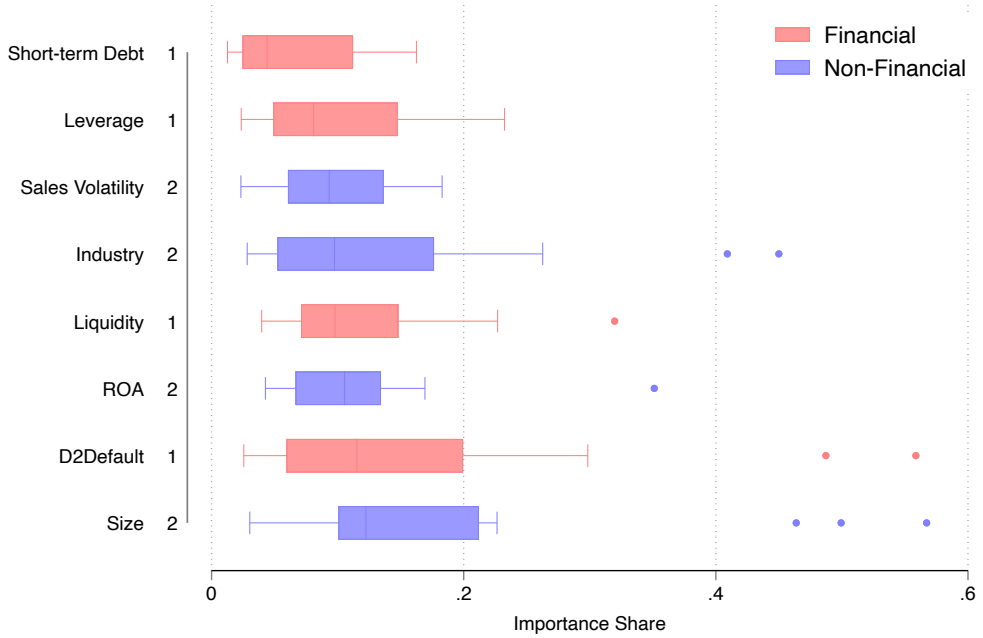
We measure the importance of each characteristic in shaping the overall heterogeneity in firms' sensitivity by examining their contribution to the moment function, which is based on the proportion of splits related to the characteristic of interest. In other words, the characteristic importance measure represents a depth-weighted average of the number of splits associated with the characteristics of interest. This provides an intuitive measure that can be interpreted as the percentage of heterogeneity in sensitivities due to a certain firm characteristic. We compute the share of importance for each characteristic in each aggregate shock - outcome pair, and then aggregate these contributions across all pairs by characteristic. Figure 2 presents the relative importance of each characteristic in generating variations in sensitivity across all aggregate shock-outcome variable pairs. Figures 14 to 15 in Appendix C report the share of importance of each firm characteristic in all distinct cases.

Share of importance across all cases Figure 2 shows that, on average across all aggregate shock-outcome variable pairs, firms' size and distance-to-default have the most significant impact on shaping the heterogeneity in firms' sensitivity among all the characteristics included in the model. On average, firms' size accounts for 19% of the heterogeneity in the response to aggregate shocks across all cases, while distance-to-default has a share of importance of 16%, with both having peaks up to almost 60%. More generally, across all aggregate shock-outcome variable pairs together, non-financial characteristics demonstrate greater importance in explaining firms' sensitivity to aggregate fluctuations as they rank higher than financial characteristics on average.[17] The top two non-financial characteristics, firm size and return on assets, rank first and third, respectively. In comparison, the top two financial characteristics, distance-to-default and liquidity, rank second and fourth, respectively.

On average, importance shares are dispersed across many characteristics, and within each characteristic, there is significant heterogeneity in rankings across all aggregate shock-outcome variable pairs. These patterns suggest that different characteristics, or combinations thereof, hold relevance across various aggregate shock-outcome variable pairs. In line with this, Table 6 in Appendix C shows that there is no statistically significant correlation across shocks in how the shares of importance are distributed across firm characteristic, while there is some positive correlation in how the shares of importance are distributed across real outcome variables (sales, investment rate, and debt rate). Consistently, Figure 3 shows that non-financial characteristics are predominant in explaining the heterogeneity in sensitivities to business cycles across all outcome variables (Crouzet and

[17] On average, non-financial characteristic all together account for 55% of the heterogeneity in the response to aggregate shocks across all cases, with financial characteristic accounting for the remaining 45%.

Figure 2: Importance share based on financial and non-financial characteristics

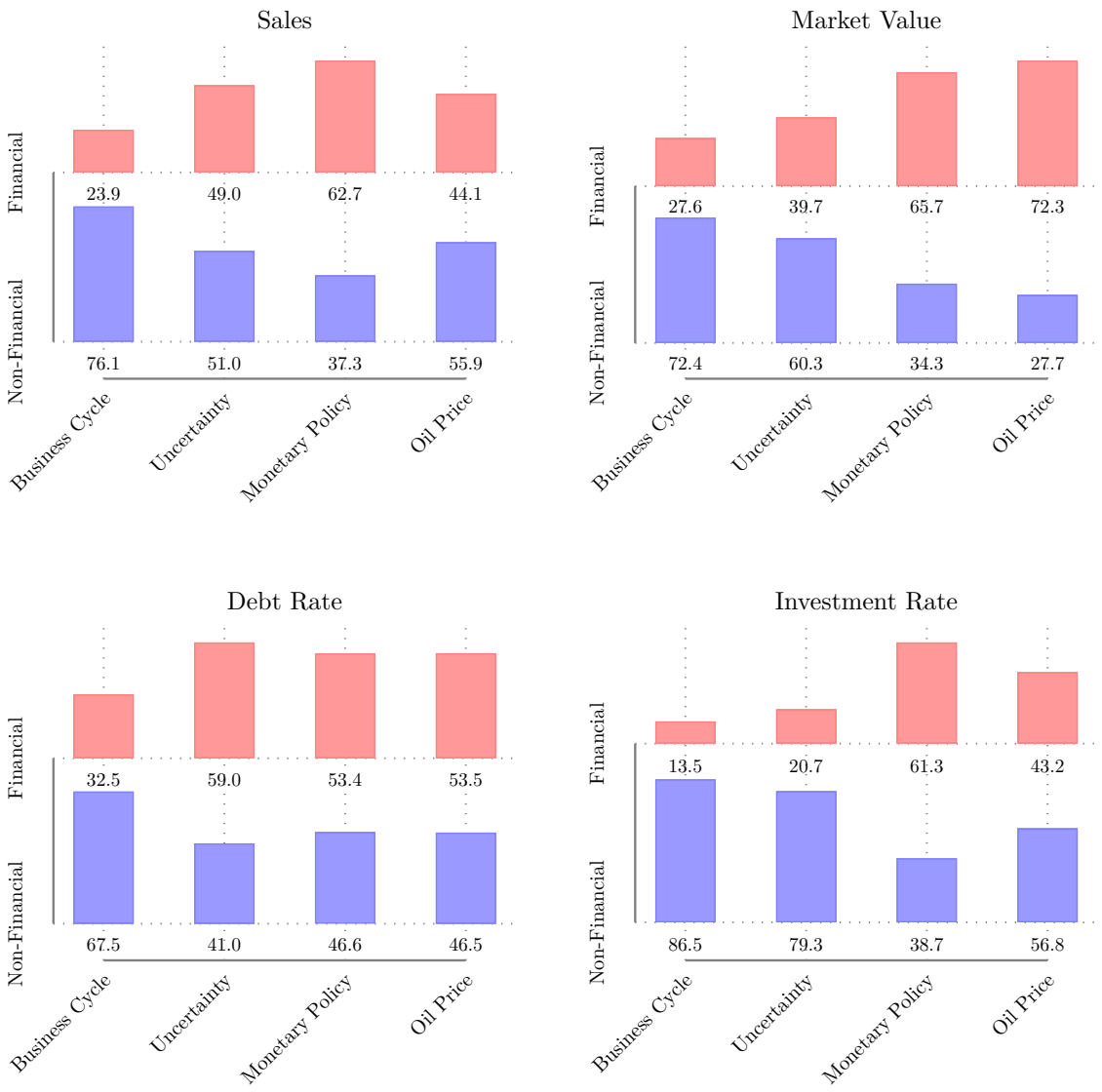


Notes: The figure displays the distribution of importance shares for firms' balance sheet characteristics across all aggregate shock-outcome variable pairs, ordered in decreasing order of importance share based on the median pair. The "Financial" group (red) consists of four variables that capture firms' financial conditions: leverage, liquidity ratio, distance to default, and short-term debt ratio. The "Non-Financial" group (blue) consists of four variables that capture firms' size, profitability, and industry: total assets (log), sales growth volatility, return on assets, and industry classification at the 5-digit NAICS level. The box plot shows the 75% importance share across all combinations of the pairs. Dotted points represent individual pairs.

Mehrotra, 2020). Similarly, financial characteristics matters relatively more in the response of all outcome variables to monetary policy (Ottonello and Winberry, 2018). No clear pattern emerges between financial and non-financial characteristics as the most crucial factors in explaining heterogeneity in response to changes in uncertainty and oil price shocks: non-financial characteristics matter more for the response of market value and investment to uncertainty shocks, while financial variables are more relevant than non-financial for the response of market value to oil shocks.

Relation to the literature Given the relevance of different combinations of characteristics depending on the aggregate shock - outcome variable pair considered, we provide additional details focusing on specific cases previously studied in the literature. First, non-financial characteristics are primarily relevant for explaining firms' sensitivity to business cycle fluctuations, with a peak of 86.5% for investment and 76% for sales. Figure 14 in the Appendix shows that, within non-

Figure 3: Importance Share by outcome variable



Notes: The figure plots the importance share by group of firms' balance sheet characteristics across all aggregate shock-outcome variable pairs. The importance share for the group "Financial" (red bars) is constructed by summing the contributions of four variables the capture firms' financial conditions: leverage, liquidity ratio, distance to default, and short-term debt ratio. The importance share for the group "Non-Financial" (blue bars) is constructed by summing the contributions of the other four variables that capture firms' size, profitability and industry: total assets (log), sales growth volatility, return on assets, and industry classification based on 5-digit NAICS level. The grouping follow standard practice in the main literature that study firm heterogeneity and aggregate fluctuations.

financial characteristics, industry stands out as the most important determinant for sales growth, accounting for 45% of the importance share, while size is particularly relevant for investment and debt issuances (approximately 50%), and overall profitability for market value (35%). These results are consistent with prior research by Crouzet and Mehrotra (2020), which suggests that demand and industry scope play significant roles as key determinants of firms' heightened sensitivity to business cycle fluctuations.

Secondly, firms' financial conditions become particularly relevant when examining the heterogeneous sensitivity of firms' observables in response to identified monetary policy shocks. Figure 16 in the Appendix C shows that financial characteristics, especially distance to default and liquidity, are important to explain the firm-level sensitivity to monetary policy shocks. In partcular, extensive literature focus on the role of firm characteristics on the response of investment to monetary policy. We find that liquidity and distance-to-default have a strong and quantitative similar impact on the heterogeneity in the response of investment to monetary policy (23% and 21%, respectively). This is in line with the competing empirical finds of both Ottonello and Winberry (2018), which focus on distance-to-default, and Jeenas (2018a), which focuses on liquidity. Similarly, the heterogeneous response of debt to monetary policy is largely driven by financial characteristics, with leverage (17%) and short-term debt (16%) being the most influential, which aligns with economic intuition. The heterogeneous response of firms' stock market prices to unexpected interest changes is primarily driven by differences in firms' default probability, accounting for 56% of the variation.

Thirdly, Figure 15 in the Appendix C shows that non-financial characteristics, particularly size and industry, are relevant for the heterogeneity in the response of investment and stock market prices to changes in uncertainty (57% and 41%, respectively). In relation to the literature studying the role of financial positions for uncertainty shocks (Alfaro et al., 2024), we find that financial characteristics, particularly distance-to-default, leverage, and liquidity are relevant only for the response of debt issuance and, to a certain extent, for the response of sales, which are not the focus of their work.

Lastly, relatively few literature has explored the effect of oil shocks and its heterogeneity across firms. We add to the literature showing that financial variables such as leverage and liquidity are key for the heterogeneity in the response of market value to oil shocks, with an importance share of 32% and 22%, respectively. On the contrary, real outcome variables (sales, investment, and debt) have a more balanced division between financial and non-financial characteristics and a more dispersed allocation across characteristics, suggesting that these cases require a more comprehensive analysis.

Table 1: Test on non-linearity of CATEs

	p-value HC Test		EDF GAM	
	< 0.01	< 0.05	> 1	> 2
Sales				
All Variables	0.79	0.89	0.89	0.93
Importance Share > 10%	0.80	1.00	0.83	0.89
Market Value				
All Variables	0.93	0.96	1.00	1.00
Importance Share > 10%	0.85	0.92	1.00	1.00
Debt Rate				
All Variables	0.96	0.96	0.82	0.86
Importance Share > 10%	0.94	0.94	0.91	0.91
Investment Rate				
All Variables	0.96	0.96	0.89	0.96
Importance Share > 10%	0.93	0.93	0.93	1.00

Notes: The table reports a summary of the Harvey - Collier test for linearity in the relationship between covariates and the CATE produced by the Causal Forest.

Fact 3: Sensitivity to aggregate shocks is non-linear in firms' characteristics.

One of the key limitations of a standard OLS model as in Equation (1) is the implicit assumption of linearity in the interaction term. This translates into the heterogeneity in sensitivity being linearly dependent on each firm characteristic, ruling out richer specifications for the CATEs. The most common alternatives are introducing quadratic terms or estimating the interaction term non-parametrically after arbitrarily splitting the sample along the specific dimension of interest (Crouzet and Mehrotra, 2020; Cloyne et al., 2018). However, these alternatives also fall short in fully accounting for the heterogeneity in sensitivity compared to the casual forest model in Equation (2), which does not assume any functional form and its objective is to maximize the estimated heterogeneity.

Testing for linearity We show that firms' sensitivities to aggregate fluctuations, estimated via causal forest, exhibit non-linearity with respect to their balance sheet characteristics. Specifically, we examine the relationship between each covariate X_i and the Conditional Average Treatment Effect (CATE) generated by the Causal Forest Algorithm. To formally test whether the relationship is linear at the micro-level sensitivity, we conduct a Harvey-Collier test for linearity in the relationship between the firms' characteristics and the firms' micro-level sensitivity estimated using GRF. The test involves a t-test on the mean of the recursive residuals between the micro-sensitivities and the

covariates, which should be equal to zero under the null hypothesis that the relationship is linear. We perform the test for each characteristic-aggregate shock-outcome variable tuple. Columns (1) and (2) of Table 1 report the share of cases for each outcome variable in which we reject the linearity hypothesis at the 1% and 5% significance levels, respectively.[18] Across all outcome variables, 90% of cases strongly reject the linearity hypothesis, with no particular difference when we condition to the most relevant characteristics (those characteristics with an importance share larger than 10%).

As a robustness check, we estimate a Generalized Additive Model (GAM) of the firms' sensitivity on firms' characteristics. A GAM is a generalized linear model in which a univariate response variable depends linearly on unknown smooth functions of some predictor variables. The advantage of estimating a GAM is twofold. The effective degrees of freedom (EDF) estimated from generalized additive models are used as a proxy for the degree of non-linearity: an EDF of 1 is equivalent to a linear relationship, while an EDF > 1 indicates a non-linear relationship. Columns (3) and (4) of Table 1 report the share of cases for each outcome variable in which the EDF is greater than 1 and 2, respectively. In this case, more than 90% of cases estimate a non-linear relationship between sensitivities and firm characteristics.[19]

Moreover, we compare the predictive performance of a GAM to that of a standard linear model for the relationship between firms' sensitivity and firm characteristics. Table 7 in Appendix C shows that, using the AIC and the adjusted R^2, a non-linear relationship between firms' sensitivity and characteristics performs better in almost all cases than a linear model. This suggests the restrictiveness of the linearity assumption and the importance of departing from it.

Relation to the literature We discuss a set of notable examples from the literature to better explain and further support this point. Specifically, we study the relationship between the estimated micro sensitivities to aggregate fluctuations and selected firms' characteristics in the linear benchmark case, and compare the results with those of piecewise linear regressions that allow for breakpoints ψ over firms' characteristics:

$$\mathbb{E}\left(\beta(X_{i,t})|X_{i,t}=x\right)=\beta_0+\beta_1 X_{i,t}+\beta_2\left(X_{i,t}-\psi\right)_+ +\epsilon_{i,t} \tag{4}$$

where $\mathbb{E}\left(b_{i,t}|X_{i,t}=x\right)$ is the expected value of the firm-level sensitivity $b_{i,t}$ to aggregate fluctuations, conditional on covariates $X_{i,t}$ from the GRF. $\left(X_{i,t}-\psi\right)_+ = \left(X_{i,t}-\psi\right)\times\mathbf{1}\left(X_{i,t}>\psi\right)$ is a piecewise linear term that is zero when $X_{i,t}\leq\psi$ and $X_{i,t}-\psi$ when $X_{i,t}>\psi$. The coefficients of interest are β_1, which captures the change in sensitivity associated with a one-unit change in $X_{i,t}$ before the

[18] Table 8 in Appendix C reports the p-values for all individual characteristic-aggregate shock-outcome variable tuples.

[19] Table 9 in Appendix C reports the estimated EDF for all individual GAMs estimated for each characteristic-aggregate shock-outcome variable tuple.

Table 2: Linear and non-linear heterogeneity in CATEs to aggregate fluctuations

Relevant Literature	Independent Variable	Dependent Variable	Linear CATE Benchmark	Coefficient β_1	Coefficient β_2	Breakpoints Percentile (Value)
Panel A. Business Cycle						
Crouzet and Mehrotra (2020)	Sales	Size	0	0.19	-0.27	44(0.22)
	Investment	Size	-0.17	-0.2	0.21	89(3.6)
	Market Value	Size	0.2	0.08	0.28	68(1.79)
Covas and Haan (2011)	Debt	Size	-0.21	0.21	-0.71	54(0.89)
Bernanke et al. (1999)	Sales	Distance to Default	-0.04	-0.04	0.1	98(17.21)
	Investment	Distance to Default	-0.01	-0.01		
	Market Value	Distance to Default	0.04	0.1	-0.12	71(7.44)
	Debt	Distance to Default	-0.04	0.08	-0.15	43(4.09)
Panel B. Monetary Policy						
Ottonello and Winberry (2018)	Investment	Distance to Default	-0.08	-0.66	0.69	38(3.59)
	Investment	Leverage	3.12	12.81	-11.7	33(0.16)
Jeenas (2018b)	Investment	Liquidity	-4.56	-5.14	5.5	96(0.6)
	Investment	Short-Term Debt	-1.17	-2.25	2.45	70(0.42)
Gertler and Gilchrist (1994)	Investment	Size	0.18	-0.71	1.1	34(-0.49)
Panel C. Uncertainty						
Alfaro et al. (2024)	Investment	Size	0.03	0.04	-0.04	91(3.8)
	Investment	Distance to Default	0	0	0.01	29(2.79)
	Debt	Size	0.03	-0.01	0.05	40(-0.08)
	Debt	Distance to Default	0.01	-0.02	0.03	44(4.13)
	Market Value	Size	0.01	-0.11	0.13	30(-0.72)
	Market Value	Distance to Default	0.02	0.03	-0.02	94(13.95)

Notes: This table presents the results of segmented regression models examining the non-linearities and threshold effects in firm sensitivity to aggregate fluctuations and firms' characteristics, based on notable examples from the literature. The first three columns list relevant studies, along with the independent and dependent variables analyzed. The fourth column reports the coefficient of a linear regression of the firm-level sensitivity, as estimated from the GRF algorithm, on the dependent variable using a linear model. The last three columns respectively present the coefficients β_1 and β_2 from the segmented regression model, as well as the estimated breakpoints (percentiles) and their respective values in parentheses. Panel A focuses on business cycle effects, Panel B on monetary policy, and Panel C on uncertainty.

breakpoint ψ, and β_2, which is the non-linear term and captures the change in sensitivity when $X_{i,t}$ exceeds the breakpoint ψ. If the breakpoint does not exist, the difference-in-slopes parameter must be zero. Table 2 presents the estimated coefficients in relation to the most relevant cases in the literature.

Panel A of Table 2 summarizes findings from two key strands of literature: one studying the role of size on firms' sensitivity to business cycle fluctuations in sales, investment, debt issuance, and stock prices (Crouzet and Mehrotra, 2020; Covas and Haan, 2011), and the other relating to the financial accelerator theory (Bernanke et al., 1999). We highlight two main points. First, differently from Crouzet and Mehrotra (2020) which predict a monotonically decreasing relationship between sensitivity and size, we show that sales and debt sensitivity decrease with size beyond the 44th and 54th percentiles, respectively, but increase below these thresholds[20] Firms' investment sensitivity to

[20]These results fail to be captured in a linear model. A linear model fails in capturing these patterns,

the business cycle consistently decreases with size, in line with Crouzet and Mehrotra (2020), even though we show that the relationship flattens out above the 90th percentile (i.e., no heterogeneity beyond this threshold). Additionally, relatively small firms (below the 68th percentile) do not show excess stock price sensitivity to the business cycle, whereas larger firms exhibit significantly higher stock market sensitivity as size increases. Second, the relationship between firms' sensitivities and distance to default is consistent with the financial accelerator theory. A linear model would predict that firms with low default risk (or high distance to default) tend to be less sensitive to business cycle fluctuations in terms of sales, investment rates, and debt issuances, while displaying higher sensitivity in terms of stock prices. However, only the relationship in terms of sales and investment rates is well captured by a linear model. In contrast, stock price sensitivity to the cycle generally increases with distance to default, confirming that the stock prices of larger, safer firms react more strongly to business cycle fluctuations, but it flatten out for very safe firms (i.e., with distance to default greater than 7.5). Firms carrying medium-risk, on the other hand, are the ones that issue relatively more debt over the cycle (i.e., debt sensitivity to the cycle increases for firms with a distance to default lower than 4 but decreases beyond this threshold). The average CATEs for all outcome variables in relation to balance sheet characteristics, in response to business cycle fluctuations, are shown in Figure 18 in the Appendix.

Panel B of Table 2 presents results from the literature examining the impact of monetary policy on firms' investment behavior, focusing on various financial characteristics. Recent studies have primarily investigated the response of investment to monetary policy hikes and the role of leverage, default risk, size, and liquidity (Gertler and Gilchrist, 1994; Ottonello and Winberry, 2018; Jeenas, 2018b). First, consistent with the literature, firms with low default risk, low leverage, high liquidity, and high short-term debt maturity are relatively more sensitive to monetary policy shocks. However, extending the evidence from Ottonello and Winberry (2018) and Jeenas (2018b), we show the presence of a significant degree of heterogeneity in the response of investment, mainly arising from the lower end of the distribution. For instance, investment sensitivity increases markedly for firms up to the 38th percentile of distance to default and the 70th percentile of short-term debt. However, firms with a distance to default greater than 3.8 (i.e., above the 38th percentile of the distribution) and short-term debt above 42% are equally negatively affected by monetary policy[21]. Figure 20 in the Appendix shows that average response of firms' outcome variables to monetary policy shocks in relationship to balance sheet characteristics.

Finally, Panel C in Table 2 focuses on the sensitivity of investment, debt, and stock market responses to uncertainty shocks, depending on default risk and size. We highlight two results.

predicting excess sensitivity of smaller firms in debt issuance over the cycle (linear benchmark is -0.21) and virtually no effect of size on sales.

[21] A similar pattern is observed when studying the relationship between the elasticity of investment to interest rates and leverage, liquidity, and size.

First, consistent with previous literature, small, non-profitable, and high-default-risk firms tend to divest and deleverage relatively more in response to uncertainty shocks, reminiscent of the financial multiplier mechanism described in Alfaro et al. (2024). During periods of exogenous spikes in uncertainty, the stock market prices of larger and less risky firms tend to be shielded from these spikes, making them less sensitive to uncertainty shocks. Second, the heterogeneous effects of uncertainty shocks tend to be non-linear over characteristics depending on specific cases. For instance, firms that are very large in size (above 90th percentile), display almost no sensitivity of real investment and debt rate to spike in uncertainty shocks. Instead, debt issuance rates are U-shaped over size and default risk suggesting that firms' that likely have lost their financial stability (distance to default close to 0 or are very small) are no able to adjust their debt positions as much as firms that carry slightly medium default risk. Overall, those results are in line with previous findings in the literature. The average CATEs for all outcome variables in relation to balance sheet characteristics, in response to uncertainty shocks, are shown in Figure 19 in the Appendix.

Sensitivity to oil prices depending on balance sheet characteristics is less studied in the literature. Overall, we find that balance sheet characteristics are a major driver behind stock market sensitivity. In general, oil price shocks seem to affect firms with high leverage, low cash reserves, and higher default risk more strongly, with the average effect flattening out on the right-hand side of the distributions. Non-financial characteristics (size, sales volatility, and growth) are important for understanding the micro-response of real investment and debt issuances to exogenous oil price shocks. The response of all outcome variables to oil price shocks, reflected in the average CATEs across balance sheet characteristics, is presented in Figure 21 in the Appendix.

4 Aggregate Implications

This section studies the aggregate implication of the heterogeneity in firms' sensitivity to aggregate shocks. We first propose a theory of aggregation to compute the response of any aggregate variable to aggregate fluctuations by aggregating up firm-level individual responses. Then, using this aggregation rule, combined with counterfactual sensitivities obtained from the estimated random forests, we assess the contribution and origins of the heterogeneity in firms' sensitivity to aggregate fluctuations."

Decomposition and Aggregation - Theory. Consider a set I_t of firms continuing to operate between t and $t-1$. Let G_t and $g_{i,t}$ denote the aggregate and the firm-level response of variable Y_t following an aggregate shock W_t, respectively:

$$G_t = \frac{Y_t}{Y_{t-1}} \qquad g_{i,t} = \frac{Y_{i,t}}{Y_{i,t-1}}. \tag{5}$$

Let ω_{it-1} be the share of Y_{t-1} accounted for by firm i:

$$\omega_{it-1} = \frac{Y_{i,t-1}}{Y_{t-1}} \qquad \text{where } Y_{t-1} = \sum_{i \in I_t} Y_{i,t-1} \tag{6}$$

It follows that we can write the aggregate response of variable Y_t to an aggregate shock at time t as:

$$G_t = \sum_{i \in I_t} \omega_{i,t-1} g_{i,t}. \tag{7}$$

Importantly, in our setting, we can retrieve firm-level growth rate from the estimated random forests as $\widehat{g_{i,t}} = \widehat{\beta\,(x)} W_t$, and calculate the relative shares from our dataset.

The aggregation in Equation (7) highlights that both firms' individual sensitivities and shares matter for the aggregate response. In fact, we can write the following decomposition:

$$G_t = \overline{g_t} + \mathbb{C}\text{ov}(w_{i,t-1}, g_{i,t}), \tag{8}$$

where the first term is the average sensitivity of $g_{i,t}$ across firms, $\frac{1}{|I_t|} \sum_{i \in I_t} g_{i,t}$, and the second term is the covariance between firm sensitivity and firms' importance in the aggregate, $\sum_{i \in I_t} \left(\omega_{i,t-1} - \frac{1}{|I_t|} \right) (g_{i,t} - \overline{g_t})$.

We use Equations (7) and (8) to construct and decompose counterfactual aggregate responses in three steps. First, for any aggregate shock-outcome variable pair, we leverage the estimated random forests to generate counterfactual firm-level sensitivities, β_{it}^{cf} and $g_{i,t}^{cf}$, under difference scenarios. We use Equation (7) to construct the counterfactual aggregate response G_t^{cf}. Second, following Crouzet and Mehrotra (2020) we use the following time-series regression to estimate the average aggregate effects of an aggregate shock over time:

$$Z_t = \alpha + \gamma W_t + \epsilon_t, \tag{9}$$

where Z_t can be a counterfactual aggregate response, G_t^{cf}, or our benchmark aggregate response, G_t, and γ represents our coefficient of interest capturing the average aggregate effects of a aggregate shock W_t. Lastly, we apply our decomposition to assess the relative importance of the mean and covariance terms and gain more insights into the drivers of the aggregate response. Specifically, we estimate the econometric specification in Equation (9) using the mean or the covariance terms from Equation (8) as dependent variable Z_t. It is easy to show that, given $G_t = \overline{g_t} + \mathbb{C}\text{ov}(w_{i,t-1}, g_{i,t})$, we have that $\gamma = \gamma_1 + \gamma_2$, where the coefficients are estimated from the following specifications: $G_t = \alpha + \gamma W_t + \epsilon_t$, $\overline{g_t} = \alpha + \gamma_1 W_t + \epsilon_t$, and $\mathbb{C}\text{ov}(w_{i,t-1}, g_{i,t}) = \alpha + \gamma_2 W_t + \epsilon_t$, respectively.

Constructing counterfactual scenarios. Our benchmark scenario is the one that considers the whole distribution of estimated firm-level sensitivities, $\widehat{\beta(x)}$, in constructing the aggregate response G_t. We leverage the estimated random forest to predict counterfactual firm-level sensitivities and assess the origins to aggregate fluctuations. We consider three type of counterfactual exercises:

1. Heterogeneity in sensitivities. In the first exercise, we aim to assess the impact of heterogeneous firm-level sensitivity on aggregate fluctuations. To this end, we construct a counterfactual scenario by assuming that all firms in the economy share the same firm-level sensitivity. Specifically, we set the counterfactual sensitivity of each firm i at quarter t equal to the median cross-sectional sensitivity at quarter t, $\beta_{it}^{cf} = \text{median}_t(\widehat{\beta(x)})$. We then construct the counterfactual firm-level and aggregate response of each outcome variable, g_{it}^{cf} and G_t^{cf}, respectively. The difference in the estimated γ is informative on the role of heterogeneity in firm-level sensitivity for aggregate fluctuations.

2. Heterogeneity in the underlying characteristics. We explore the role of heterogeneity in the underlying firm characteristics for aggregate fluctuations by leveraging the estimated random forest and predicting counterfactual firm-level sensitivities under different distributions of firm characteristics. Comparing this counterfactual to the estimated γ from the previous exercise we are able to assess the aggregate implications of the non-linearities in the mapping between firm characteristics and sensitivity explored in Section 3. We assume that each firm i in industry j at quarter t has the same characteristics or subset of those characteristics. Specifically, we consider the case in which firms have all characteristics equal to the median value of each characteristic in industry j at quarter t, such that $\beta_{ijt}^{cf} = \beta(\text{median}_{jt}(X_{ijt}))$. In addition, given the different role of financial and non-financial characteristics shown in Section 3, we further consider the case in which firms have all financial (or non-financial) characteristics equal to the median value of each financial (or non-financial) characteristic in industry j at quarter t, such that $\beta_{ijt}^{cf} = \beta(\text{median}_{jt}(X_{ijt}^{\text{financial}}), X_{ijt}^{\text{non-financial}}))$ (or $\beta_{ijt}^{cf} = \beta(X_{ijt}^{\text{financial}}, \text{median}(X_{ijt}^{\text{non-financial}})))$.

3. State-dependent heterogeneity. In this exercise, we explore how endogenous variations in firms' characteristics influence the amplitude of business cycle fluctuations and the response to aggregate shocks during a period of recession. To this end, we set the counterfactual sensitivity of each firm i during the four years after the start of a recession equal to the firm-level average in the two years before the start of the recession. We then construct the cumulative change in the aggregate response of each outcome variable over a 17-quarter window around the start of recessions using both the benchmark and counterfactual sets of

firm-level sensitivities. We estimate the following specification:

$$\tilde{Z}_t = \alpha + \sum_{\tau=-4}^{12} \beta_\tau \mathbf{1}_{\tau=\text{Start of recession}} + \nu_t, \tag{10}$$

where $\mathbf{1}_{\tau=\text{Start of recession}}$ is a dummy equal to when a period of recession starts, and $\tilde{Z}_t$ is the cumulative change in the benchmark or counterfactual aggregate response, G_t and G_t^{cf}.[22] The difference in the estimated cumulative response is informative on the comovement between firm-level sensitivities and periods of recessions, capturing how changes in firm characteristics during periods of recessions contribute to amplify or dampen the response to aggregate shocks.

4.1 Heterogeneity Matters for Aggregate Fluctuations

We shows that heterogeneity in firms' micro-level sensitivity and characteristics plays a crucial role in driving aggregate fluctuations, with non-financial characteristics impacting more the aggregate response of the economy. We show that non-linearities and state-dependence heterogeneity instead play a limited role for the aggregate economy, except in few selected scenarios. Moreover, based on our aggregation theory, we show that differences in the covariance term in Equation (8) tend to affect more the aggregate response, suggesting the importance of joint distribution of sensitivities and firm shares.

Heterogeneity in Sensitivities In this scenario, all firms shares the same sensitivity β - the median value across all firms - in a given quarter.[23]

The Figures 4 and 5 show that abstracting away from the heterogeneity in firm sensitivity amplifies the aggregate response of the outcome variable in almost all of the cases considered, with the notable exception of the response of market value.[24] The median increase in the aggregate response is approximately 20% across all aggregate shock - outcome variable pairs, with peaks of over 100% as in the response of investment to monetary policy and uncertainty. The Figure 4 shows that the difference between the benchmark and the counterfactual γ is statistically significant in all but two pairs (the response of sales and debt ratio to monetary policy). Moreover, directly comparing the benchmark and counterfactual γ coefficients shows that the large amplification

[22] We focus on the major recession in our sample, the Great Financial Crisis of 2008. Starting dates follow the NBER classification of recession, i.e. 2007Q3.

[23] In addition, Figure 23 in Appendix C.4 shows that results are qualitatively unchanged when we consider the average, rather than the median, sensitivity within each quarter.

[24] We quantify the amplification or dampening in the counterfactual scenario as the percentage difference between the absolute value of the benchmark γ and the counterfactual γ, i.e. $\frac{|\gamma^{\text{cf}}|-|\gamma^{\text{benchmark}}|}{|\gamma^{\text{benchmark}}|}$. The amplification and dampening is therefore measured in terms of magnitude relative to the zero.

Figure 4: Role of micro-level sensitivity for aggregate response - Median β

Notes: The figure displays, for each aggregate shock - outcome variable pair, the γ coefficient estimated using Equation (9) and the benchmark aggregate response, G_t, as dependent variable (blue diamond, "Benchmark"). It also includes the γ coefficient estimated using Equation (9) and the counterfactual aggregate response, G_t^{cf}, as dependent variable (red circle, "Counterfactual"). The counterfactual aggregate response, G_t^{cf}, is constructed assuming that all firms have the same sensitivity β equal to the median value across all firms in a given quarter. The 95th percentile confidence intervals are constructed using robust standard errors.

effects (over 100%) are driven by the benchmark response being close to zero. The reason is that the estimated benchmark sensitivities are heavily distributed around an average effect close to zero with tails of opposite sign as shown in Fact I (Figure 1).

The difference between the benchmark and the counterfactual aggregate responses is driven by the covariance term in Equation (8) being set equal to zero in the counterfactual scenario.[25] The amplification of the aggregate response suggests that the covariance term has the opposite sign as the mean effect, indicating that firms with larger shares in the economy also exhibit lower sensitivities in absolute terms. Thus, by setting the sensitivity of all firms equal to the same value,

[25]The difference between the benchmark and the counterfactual aggregate responses could also be due to a different mean effect. In the benchmark, the mean effect is equal to share-weighted average β, while in the counterfactual, the mean effect is equal to the median β. The left panel of Figure **??** in Appendix C.4 shows that the counterfactual mean effect is close to the benchmark one independently of whether the counterfactual is constructed using the median β, the mean β or the median β at the sectoral level.

Figure 5: Role of micro-level sensitivity for aggregate response - Ratio β

Notes: The figure displays, for each aggregate shock - outcome variable pair, the ratio between the absolute value of the benchmark γ coefficient estimated using Equation (9) and and the counterfactual γ, i.e. $\frac{|\gamma^{\text{cf}}|-|\gamma^{\text{benchmark}}|}{|\gamma^{\text{benchmark}}|}$. Red (blue) cells correspond to cases in which the aggregate response is larger (smaller) in the counterfactual scenario, thus suggesting amplification (dampening) of aggregate fluctuations. Values are expressed as percentages, where 0.01 corresponds to 1%. Standard error are omitted.

the aggregate response is amplified because the more relevant firms now have a counterfactually higher sensitivity. Consistently, Figure 22 in Appendix C.4 shows that covariance term has the opposite sign as the mean effect, except for those cases regarding the behaviour of market value in which the aggregate response is dampened.

We show that the within and across sector heterogeneity are equally contributing to the amplification of the aggregate response. We consider the counterfactual scenario in which the sensitivity of each firms is the median β across all firms in the same industry in a given quarter. The left panel of Figure 24 in Appendix C.4 shows that the amplification effect roughly halves in all pairs, with the median increase in the aggregate response being approximately 10% across all pairs. The reason for this smaller amplification effect is that, differently from the previous counterfactual case in which all firms have the same β and there is no heterogeneity, normalizing the β by sector-quarter leaves the possibility for sectoral heterogeneity in the response to aggregate shocks. The presence of sectoral heterogeneity translates into a non-zero covariance term. The right panel of Figure 24

in Appendix C.4 shows that the sectoral heterogeneity covaries negatively with the contribution of each sector to the aggregate response, indicating that sectors with larger shares in the economy also exhibit lower sensitivities in absolute terms. The right panel of Figure **??** in Appendix C.4 compares the covariance term across the three different scenarios, showing that the covariance term in the presence of sectoral heterogeneity lies midway between zero and the covariance term in the benchmark case. Thus, both within-sector and across-sector heterogeneity in sensitivities are quantitatively relevant in shaping the aggregate response to shocks, with the two margins equally contributing across all scenarios.

Heterogeneity in Characteristics - Non-Linearities We show that the non-linearities in the relationship between firm characteristics and sensitivity to aggregate shocks documented in Fact III in Section 3 are not quantitatively relevant for the aggregate response. We compare the aggregate response in the case all characteristics are set equal to the median value within each sector-quarter, i.e. $\beta^{cf}_{ijt} = \beta\left(\text{median}_{jt}(X_{ijt})\right)$, to the case in which the sensitivity is equal to the median β within each sector-quality, i.e. $\beta^{cf}_{ijt} = \text{median}_{jt}(\beta^{benchmark}_{ijt})$.[26] Thus, the difference in the aggregate response is informative on the quantitative relevance of the non-linearity in the mapping between characteristics and sensitivity.

Figure 6 shows that the presence of non-linearities does not exhibit a clear patterns in terms of amplification or dampening effect, given that the aggregate response when characteristics are equal to the their median is larger (in absolute terms) than the aggregate response when the sensitivity is equal to the median β in half of the cases considered. Moreover, the right panel shows that the difference is statistically significant only for the response of debt to monetary policy. Despite the richness and relevance of these non-linearities for the proper quantification of micro-level sensitivities, the limited quantitative relevance of non-linearities in the relationship between firm characteristics and firm-level sensitivity for the aggregate response is suggestive that the distribution of firms over firm characteristics is concentrated away from areas with strong non-linearities.[27]

Heterogeneity in Characteristics - Financial vs Non-Financial We leverage the estimated random forest and predict counterfactual firm-level sensitivities under different distributions

[26]Notice that sector itself is included in the set of non-financial characteristics. By choosing the median at the sector(-quarter) level, we are not abstracting away from the sectoral heterogeneity. The alternative would be to arbitrarily choose a sector as reference with the risk of not being comparable across scenarios.

[27]The effect of non-linearities on the aggregate response depends not only on the distribution of firm characteristics, but also on the distribution of firm sensitivities and share. Figure 26 in Appendix C shows that both the mean and the covariance terms are similar across scenarios, suggesting that the key reason for the similarity is the distribution of firms over firm characteristics being concentrated away from areas with strong non-linearities.

Figure 6: Role of non-linearities for aggregate response

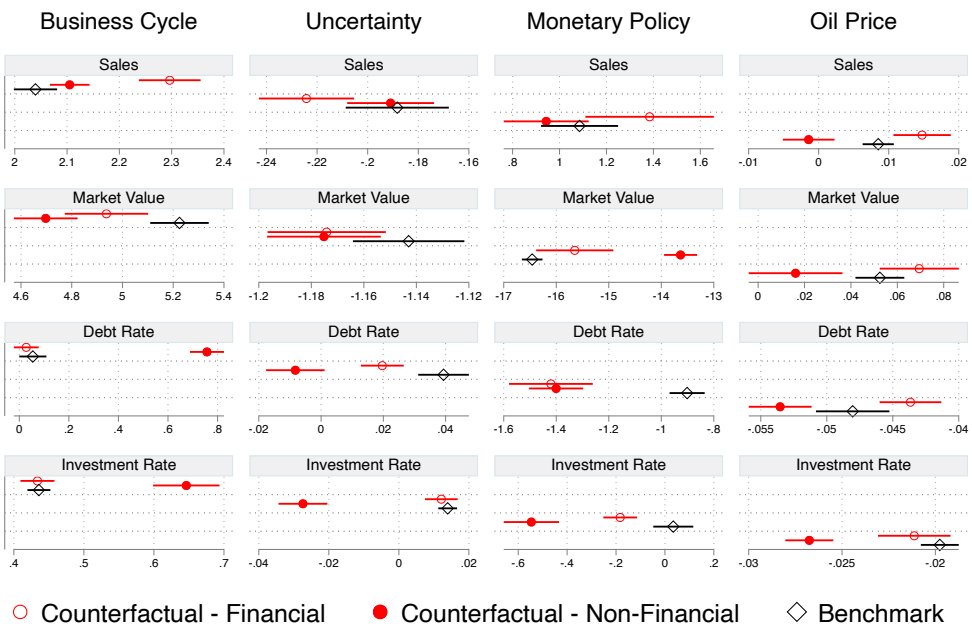
Notes: The figure plots, for each aggregate shock - outcome variable pair, the γ coefficient estimated using Equation (9) and, as dependent variable: the counterfactual aggregate response, $G_t^{\text{median } X}$ constructed assuming that firms have the same characteristics X_{it} equal to the median in each sector-quarter (blue diamond, "Counterfactual - median characteristics"); the counterfactual aggregate response, $G_t^{\text{median } \beta}$, constructed assuming that firms have the same sensitivity β equal to the median in each sector-quarter (red circle, "Counterfactual - median β"). The 95th percentile confidence intervals are constructed using robust standard errors.

of subgroups of firm characteristics. We show that abstracting away from the heterogeneity in non-financial firm characteristics generates larger departures from the benchmark aggregate response, independently of the underlying shares of importance.

We compare the aggregate response in the case non-financial characteristics are set equal to their median value within each sector-quarter, i.e. $\beta_{ijt}^{cf} = \beta\left(\text{median}_{jt}(X_{ijt}^{\text{financial}}), X^{\text{non-financial}}\right)$, and in the case in which financial characteristics are set equal to their median within each sector-quality, i.e. $\beta_{ijt}^{cf} = \beta\left(X_{ijt}^{\text{financial}}, \text{median}_{jt}(X^{\text{non-financial}})\right)$, to the benchmark aggregate response constructed with the whole set of firm-level sensitivities.

Figure 7 shows that abstracting away from the heterogeneity in non-financial characteristics generates relatively more sizeable and statistically significant departures from the aggregate response of the benchmark case than abstracting away from the heterogeneity in financial characteristics. However, the quantitatively larger role of non-financial characteristics takes place not

Figure 7: Role of firms' characteristics for aggregate response - Financial vs Non-Financial



Notes: For each aggregate shock - outcome variable pair, we report the γ coefficient estimated using Equation (9) and the benchmark aggregate response, G_t, as dependent variable (blue diamond, "Benchmark"). It also includes the γ coefficient estimated using Equation (9) and the counterfactual aggregate response, $G_t^{\text{financial}}$, as dependent variable (red hollow circle, "Counterfactual - Financial"). The counterfactual aggregate response, $G_t^{\text{financial}}$, is constructed assuming that all firms have the same financial characteristics equal to the median value across all firms in a given industry-quarter. It also includes the γ coefficient estimated using Equation (9) and the counterfactual aggregate response, $G_t^{\text{non-financial}}$, as dependent variable (red full circle, "Counterfactual - Non-Financial"). The counterfactual aggregate response, $G_t^{\text{non-financial}}$, is constructed assuming that all firms have the same non-financial characteristics equal to the median value across all firms in a given industry-quarter. The 95th percentile confidence intervals are constructed using robust standard errors.

only when non-financial characteristics are overwhelmingly relevant for the heterogeneity in firm-level responses, but also in aggregate shock - outcome variable pairs in which the role of financial characteristics is predominant. For instance, as shown in Fact II in Section 3, the heterogeneity in non-financial characteristics impacts the aggregate response more than the heterogeneity in financial characteristics in the response of investment rate and debt to business cycle, where the share of importance of non-financial characteristics is overwhelming (86% and 67%, respectively). However, the role of the heterogeneity in non-financial characteristics is stronger also when the share of importance of financial characteristics is above 60%, such as in the response of market value and investment rate to monetary policy (65% and 61%, respectively).

The key implication is that large shares of importance at the micro-level do not necessarily

translate into larger relevance for the aggregate response. The reason is that the aggregate response depends also on how the distribution of firms' share correlates with the underlying distribution of characteristics and sensitivities. In line with this, Figure 27 in Appendix C shows that, while the mean term of the aggregate response does not change when abstracting away from either financial or non-financial characteristics, most of the adjustment comes from the covariance term. This indicates the presence of a stronger covariance between firms with large shares in the economy and the underlying non-financial characteristics.

State Dependent Heterogeneity State-dependency in aggregate transmission has garnered particular interest in recent literature, for instance in studies assessing the effectiveness of monetary policy during periods of recession (Ottonello and Winberry, 2018; Jeenas, 2018a). The concern arises from the fact that, as the distribution of firms over their characteristics changes during a recession, and firms respond differently to aggregate fluctuations or shocks, policy interventions may be more or less effective due to these variations.

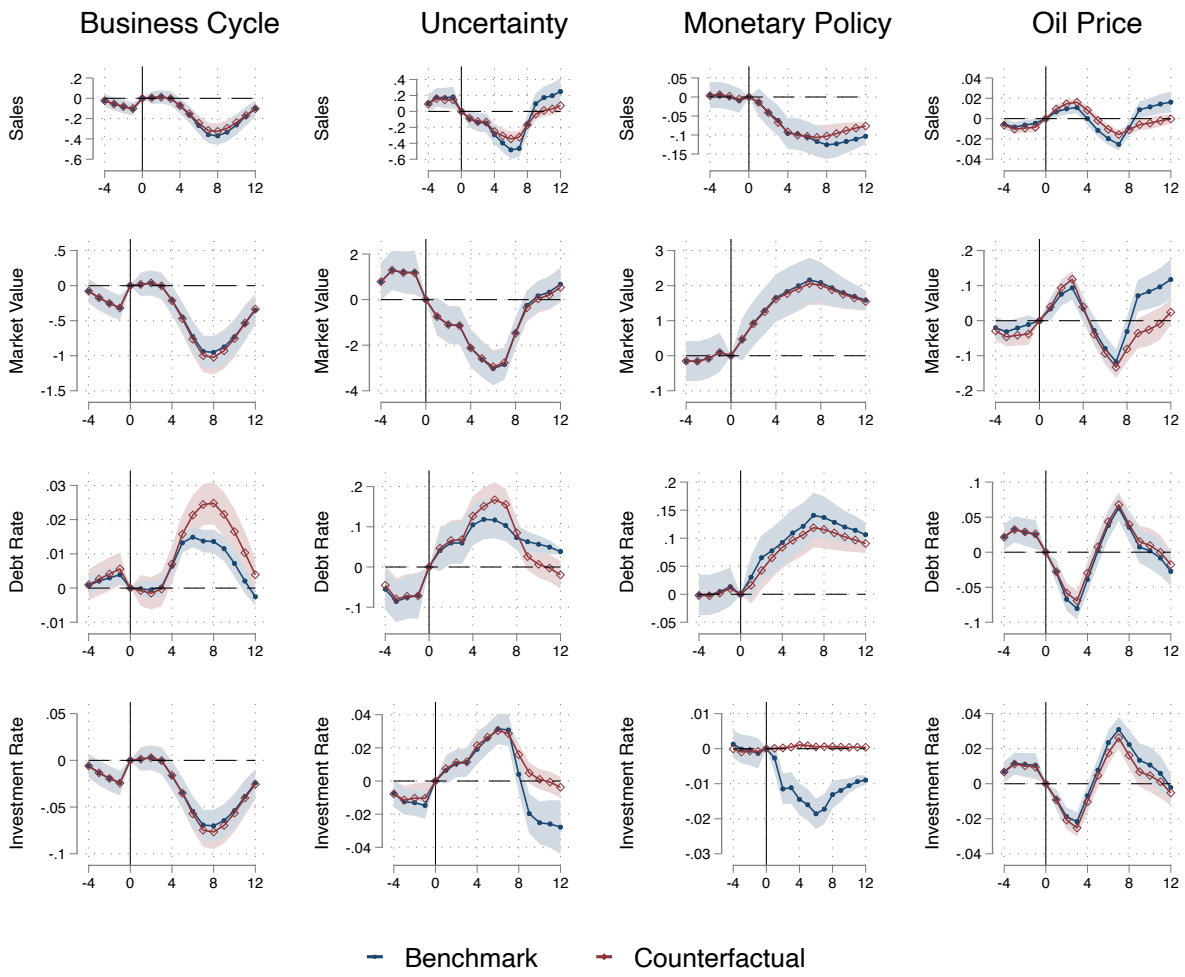
Figure 8 shows that state-dependent heterogeneity is not quantitatively relevant in shaping the response to aggregate shocks, despite statistical differences in the distribution of the underlying characteristics. Figure 8 reports the changes relative to the recession start date, capturing the cumulative path of the outcome variable estimated from the specification in Equation (10). In the counterfactual scenario we assume that the sensitivity of a firm is equal to the firm average in the two years before the start of the recession, while in the benchmark we use the whole set of firm-level sensitivities.[28] In all aggregate shock - outcome variable pairs but in the response of investment to monetary policy, we can not observes quantitatively and statistically significant differences between the benchmark and the counterfactual responses, both in the short and medium run. Notably, in line with the literature, investment would decrease less during a recession if the distributions of sensitivities is equal to the pre-crisis period, suggesting that the firms' financial deterioration amplifies the negative response of investments.[29]

This suggests that the role of shifts in the distributions of sensitivities and characteristics during periods of recessions seems to be largely overstated, and abstracting away from state-dependency heterogeneity does not bias the aggregate response to shocks. The key reason is that the shifts in the distribution of firms' characteristics in periods of recession are not economically meaningful at

[28] As robustness, we look at the case in which the underlying characteristics are kept constant to their pre-recession value. Figure 28 in Appendix A shows that the key qualitative message is not affected.

[29] Figure 30 in Appendix A shows that the covariance term is the main driver of the state-dependence effect in the short-run, while the mean term is the driver in the medium-run. This suggests that more relevant (larger shares) firms are those that face stronger state-dependency in the short-run, while the whole distribution of sensitivities and the mean effect shift in the medium-run. In all other cases, the negligible effects of state-dependence heterogeneity are due to the similarity of both the mean and covariance terms across scenarios.

Figure 8: Aggregate response to aggregate shocks in periods of boom vs bust



Notes: For each aggregate shock - outcome variable pair, we plot the estimated cumulative response of the outcome variable (relative to the start of the recession) using Equation (9). The benchmark aggregate response, G_t, as dependent variable (blue circle, "Benchmark") uses the whole set of firm-level sensitivities. The counterfactual aggregate response, G_t^{cf}, as dependent variable (red diamond, "Counterfactual") is constructed assuming that the sensitivity of a firm is equal to the firm average in the two years before the start of the recession. The 95th percentile confidence intervals are constructed using robust standard errors.

the aggregate despite being statistically significant. Figure 12 in Appendix A plots the distribution of firm characteristics among firms in the years 2005-2006 (i.e., 'Non-Recession'), and the years 2008-2009 (i.e., 'Recession'). In line with the economic intuition, in periods of recession we observe a decline in liquidity, distance-to-default, and profitability, and an increase in leverage. However, among all characteristics, we can only observe substantial differences in the distribution of distance-to-default. Despite being one of the most relevant in explaining the heterogeneity in firm sensitivity (see Figure 2), the shift in the distribution during periods of recession along only this dimension is not sufficient to influence the response of aggregate variables.[30]

5 Conclusions

In this paper we highlight the importance of understanding firm sensitivity to aggregate fluctuations in macroeconomics and finance. By employing the Generalized Random Forest model, we address the limitations of traditional OLS methods, revealing significant heterogeneity in firms' responses to economic shocks. In particular, our findings demonstrate that firm characteristics, such as size and distance-to-default, play a crucial role in these responses, with non-linear relationships uncovered through the GRF model. Additionally, we demonstrate that heterogeneity in sensitivities amplifies the aggregate response of outcome variables. Our results emphasize the necessity of advanced statistical models, especially machine learning, to accurately capture the complex drivers and dynamics of firms' responses. This approach provides new insights into the transmission of macroeconomic shocks and the effectiveness of policy interventions. Future research can extend these insights by exploring cross-country differences in firms' sensitivity, considering factors like international linkages and the transmission of global shocks, supported by detailed and extensive micro-data at the national level.

[30]Figure 29 in Appendix A plots the distributions of firm-level sensitivities from the benchmark random forest in the period of recessions and non-recession. Again, despite standard tests suggest that the distributions are statistically different, the economic implications at the aggregate level are small.

References

Alfaro, I., N. Bloom, and X. Lin (2024): "The finance uncertainty multiplier," *Journal of Political Economy*, 132, 000–000.

Athey, S., J. Tibshirani, and S. Wager (2019): "Generalized Random Forests," *The Annals of Statistics*.

Bauer, M. D. and E. T. Swanson (2023): "A reassessment of monetary policy surprises and high-frequency identification," *NBER Macroeconomics Annual*, 37, 87–155.

Begenau, J. and J. Salomao (2019): "Firm financing over the business cycle," *The Review of Financial Studies*, 32, 1235–1274.

Bernanke, B. S., M. Gertler, and S. Gilchrist (1999): "The Financial Accelerator in a Quantitative Business Cycle Framework," Elsevier, vol. 1, Part 3 of *Handbook of Macroeconomics*, chap. 21, 1341 – 1393.

Cattaneo, M. D., R. K. Crump, M. H. Farrell, and Y. Feng (2024): "On binscatter," *American Economic Review*, 114, 1488–1514.

Chang, M., X. Chen, and F. Schorfheide (2024a): "Heterogeneity and Aggregate Fluctuations," *Journal of Political Economy*.

——— (2024b): "On the Effects of Monetary Policy Shocks on Earnings and Consumption Heterogeneity," .

Chernozhukov, V., M. Demirer, E. Duflo, and I. Fernandez-Val (2018): "Generic machine learning inference on heterogeneous treatment effects in randomized experiments, with an application to immunization in India," *National Bureau of Economic Research*.

Cloyne, J., C. Ferreira, M. Froemel, and P. Surico (2018): "Monetary policy, corporate finance and investment," Tech. rep., National Bureau of Economic Research.

Covas, F. and W. J. D. Haan (2011): "The cyclical behavior of debt and equity finance," *American economic review*, 101, 877–899.

Crouzet, N. and N. R. Mehrotra (2020): "Small and large firms over the business cycle," *American Economic Review*, 110, 3549–3601.

Ehrmann, M. and M. Fratzscher (2004): "Taking stock: Monetary policy transmission to equity markets," *Journal of Money, Credit and Banking*, 719–737.

Gaiotti, E. and A. Generale (2002): "Does monetary policy have asymmetric effects? A look at the investment decisions of Italian firms," *Giornale degli economisti e annali di economia*, 29–59.

Gertler, M. and S. Gilchrist (1994): "Monetary policy, business cycles, and the behavior of small manufacturing firms," *The Quarterly Journal of Economics*, 109, 309–340.

Jeenas, P. (2018a): "Firm balance sheet liquidity, monetary policy shocks, and investment dynamics," in *Technical Report*, Working paper.

——— (2018b): "Monetary policy shocks, financial structure, and firm activity: A panel approach," *Financial Structure, and Firm Activity: A Panel Approach (January 5, 2018)*.

Jordà, Ò. (2005): "Estimation and inference of impulse responses by local projections," *American economic review*, 95, 161–182.

Jurado, K., S. C. Ludvigson, and S. Ng (2015): "Measuring uncertainty," *American Economic Review*, 105, 1177–1216.

Känzig, D. R. (2021): "The macroeconomic effects of oil supply news: Evidence from OPEC announcements," *American Economic Review*, 111, 1092–1125.

Khazra, N. (2021): "Heterogeneities in the House Price Elasticity of Consumption," .

Kumar, S., Y. Gorodnichenko, and O. Coibion (2023): "The effect of macroeconomic uncertainty on firm decisions," *Econometrica*, 91, 1297–1332.

Merton, R. C. (1974): "On the pricing of corporate debt: The risk structure of interest rates," *The Journal of finance*, 29, 449–470.

Narayan, P. K. and S. S. Sharma (2011): "New evidence on oil price and firm returns," *Journal of Banking & Finance*, 35, 3253–3262.

Ottonello, P. and T. Winberry (2018): "Financial heterogeneity and the investment channel of monetary policy," Tech. rep., National Bureau of Economic Research.

Paranhos, L. (2024): "How Do Firms Financial Conditions Influence the Transmission of Monetary Policy A Non-parametric Perspective," .

Peersman, G. and F. Smets (2005): "The industry effects of monetary policy in the euro area," *The Economic Journal*, 115, 319–342.

Stock, J. H. and M. W. Watson (2018): "Identification and estimation of dynamic causal effects in macroeconomics using external instruments," *The Economic Journal*, 128, 917–948.

Tsai, C.-L. (2015): "How do US stock returns respond differently to oil price shocks pre-crisis, within the financial crisis, and post-crisis?" *Energy Economics*, 50, 47–62.

Appendix

A Construction of the dataset and cleaning

A.1 Firm-level variables

We construct the firm-level variables in the Compustat database following standard practices. Outcome variables are calculated as a 1-year percentage growth using the Haltiwanger formula. Nominal sales are represented by the variable *saleq* in Compustat. The market value of the firm is the stock price (*prccq*) multiplied by the number of outstanding shares (*cshoq*). The investment rate is the 1-year change in capital stock, with capital stock equal to the book value of capital calculated using the perpetual inventory method. The initial value of a firm's capital stock is measured as the earliest available entry of *ppegtq*, and we then iteratively construct it from *ppentq*. Debt issuances are the percentage change in total debt, calculated as the sum of debt in current liabilities (*dlcq*) and long-term debt (*dlttq*). Inventories are represented by the variable *invtq* in Compustat. Independent variables are always expressed in levels. Leverage is calculated as the ratio of debt in current liabilities (*dlcq*) and long-term debt (*dlttq*) to total assets (*atq*). The cash ratio is the ratio of cash and short-term investments (*cheq*) to total assets (*atq*). Sales growth volatility is the standard deviation of firms' real sales growth in a 10-year rolling window. Distance to default is calculated for each firm using the algorithm in Merton (1974). The short-term debt ratio is the ratio of current debt (*dlcq*) to total debt. Size is the log of total assets (*atq*). Return on assets is the ratio of net income (*niq*) to total assets. Finally, industry scope is proxied with industry classification based on the NAICS-5 industry digit. All the independent variables, with the exception of industry classification, are yearly averaged before cleaning.

Additionally, to compute variables in real terms, we deflate capital stock, sales, and total assets using the implied price index of gross value added in the U.S. non-farm business sector.

A.2 Sample selections and cleaning

The sample period is 1990Q1 to 2018Q4. We perform the following cleaning steps:

i) We keep only US-based firms, $fic_{i,t}$ ="USA".

ii) To avoid firms with strange production functions, drop regulated utilities and financial companies, we drop all firm-quarters for which the 4-digit sic code is in the range [4900,5000) or [6000,7000).

iii) To get rid of years with extremely large values for acquisitions to avoid the influence of large mergers, we drop all firm-quarters for which the value of acquisitions $acq_{i,t}$ is greater than 5% of total assets $atq_{i,t}$.

iv) We drop all firm-quarters for which the measurement of Total Assets $atq_{i,t}$, Sales $saleq_{i,t}$, Property, Plant and Equipment (Net) $ppentq_{i,t}$, Cash and Short-Term Investments $cheq_{i,t}$, Debt in Current Liabilities $dlcq_{i,t}$, Total Long-Term Debt $dlttq_{i,t}$, Total Inventories $invtq_{i,t}$ are missing or negative.

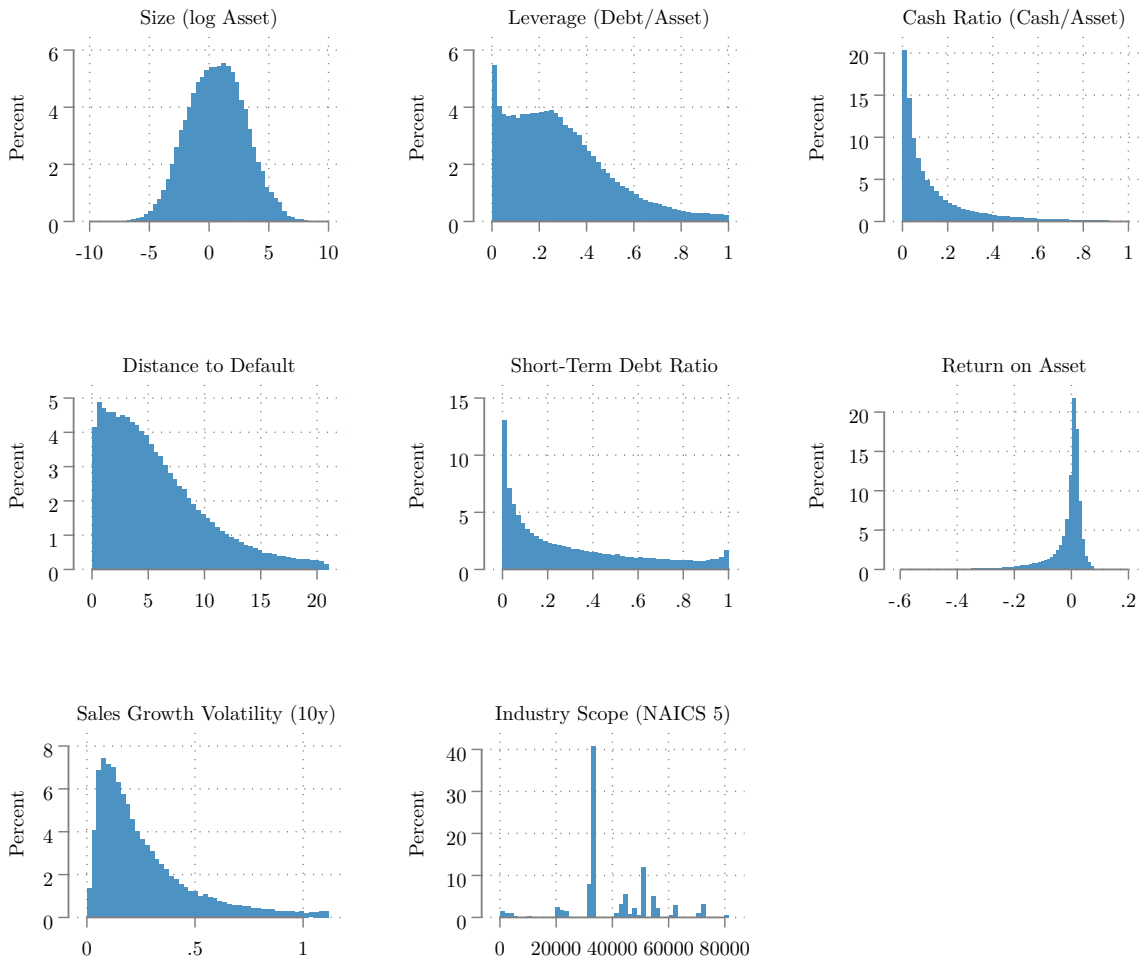
v) We drop all firm-quarters before a firm's first observation of Property, Plant, and Equipment (Gross) $ppegtq_{i,t}$.

Before estimating the models, we trim the variables at the top 1.5% level when the variables are strictly positive, and we trim 1.5% on both sides if the variables can also be negative. To reduce the number of missing values in the GRF, we linearly interpolate each independent variable after completing all cleaning steps.

We further group variables by type, distinguishing between financial and non-financial characteristics. Financial variables include leverage, liquidity ratio, distance to default, and short-term debt ratio. Non-financial variables include total assets (log), sales growth volatility, return on assets, and industry classification at the 5-digit NAICS level.

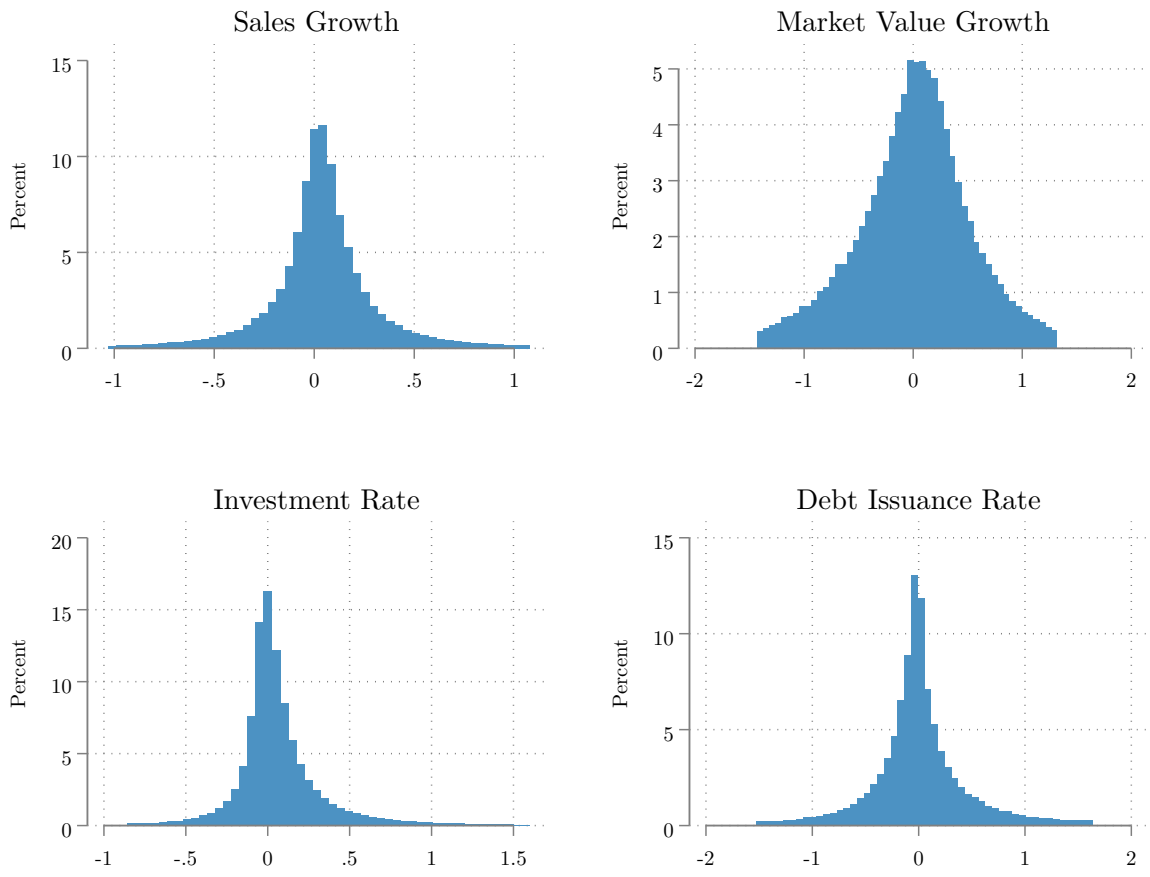
A.3 Histogram and summary statistics

Figure 9: Distribution of the independent variables



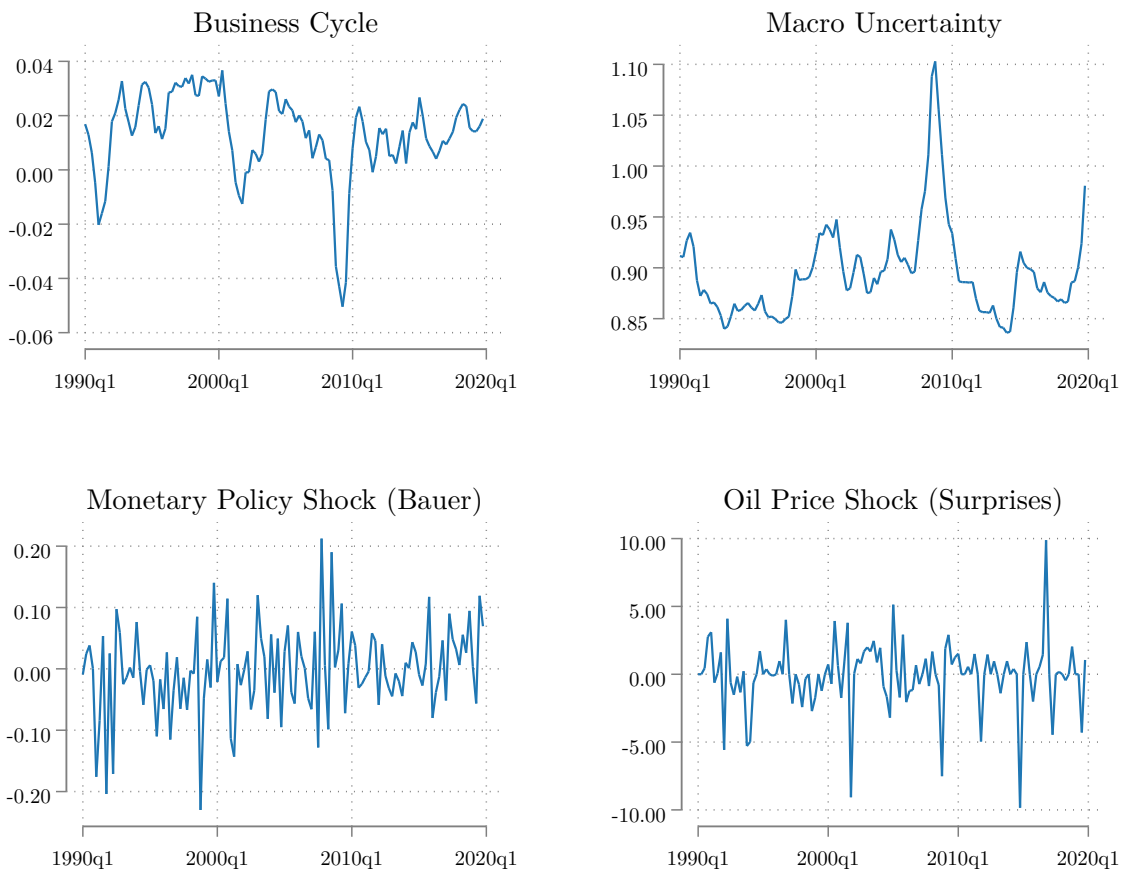
Notes: The figure shows the distribution of firm-quarter balance sheet characteristics used as independent variables in the GRF algorithm. The data are from quarterly Compustat, spanning from 1990-Q1 to 2018-Q4. Variables are trimmed at the 98.5th percentile and then linearly interpolated before being used in the algorithm. Additional details on variable construction and data cleaning are provided in Appendix A.

Figure 10: Distribution of the dependent variables



Notes: The figure shows the distribution of firm-quarter balance sheet characteristics used as dependent variables in the GRF algorithm. The data are from quarterly Compustat, spanning from 1990-Q1 to 2018-Q4. Growth rates are annual and they are calculated using the Haltiwanger formula. Variables are trimmed at the 1.5th and 98.5th percentile before being used in the algorithm. Units of measurement are in percentage points, where 0.01 represents 1%. Additional details on variable construction and data cleaning are provided in Appendix A.

Figure 11: Time series of the aggregate fluctuations



Notes: The figure shows the time-series of the aggregate fluctuations and shocks used in the paper. Units of measurement are in percentage points, where 0.01 represents 1%. Additional information on the variable construction can be found in Appendix A.

Table 3: Summary statistics

Variables	Mean	SD	Min	Max	N
Independent					
Size (log Asset)	0.626	2.409	-9.356	8.612	448856
Leverage (Debt/Asset)	0.290	0.212	0.000	1.000	339760
Cash Ratio (Cash/Asset)	0.141	0.174	0.000	0.993	363361
Distance to Default	5.763	4.453	0.000	21.027	336085
Short-Term Debt Ratio	0.302	0.289	0.000	1.000	443857
Return on Asset	-0.018	0.071	-0.456	0.077	437471
Sales Growth Volatility (10y)	0.267	0.225	0.000	1.117	378874
Dependent					
Sales Growth	0.035	0.271	-1.030	1.075	239625
Market Value Growth	0.005	0.510	-1.429	1.320	214285
Investment Rate	0.068	0.275	-0.858	1.597	418937
Debt Issuance Rate	0.005	0.451	-1.532	1.641	227627
Inventory Growth	0.017	0.306	-1.282	1.219	203697

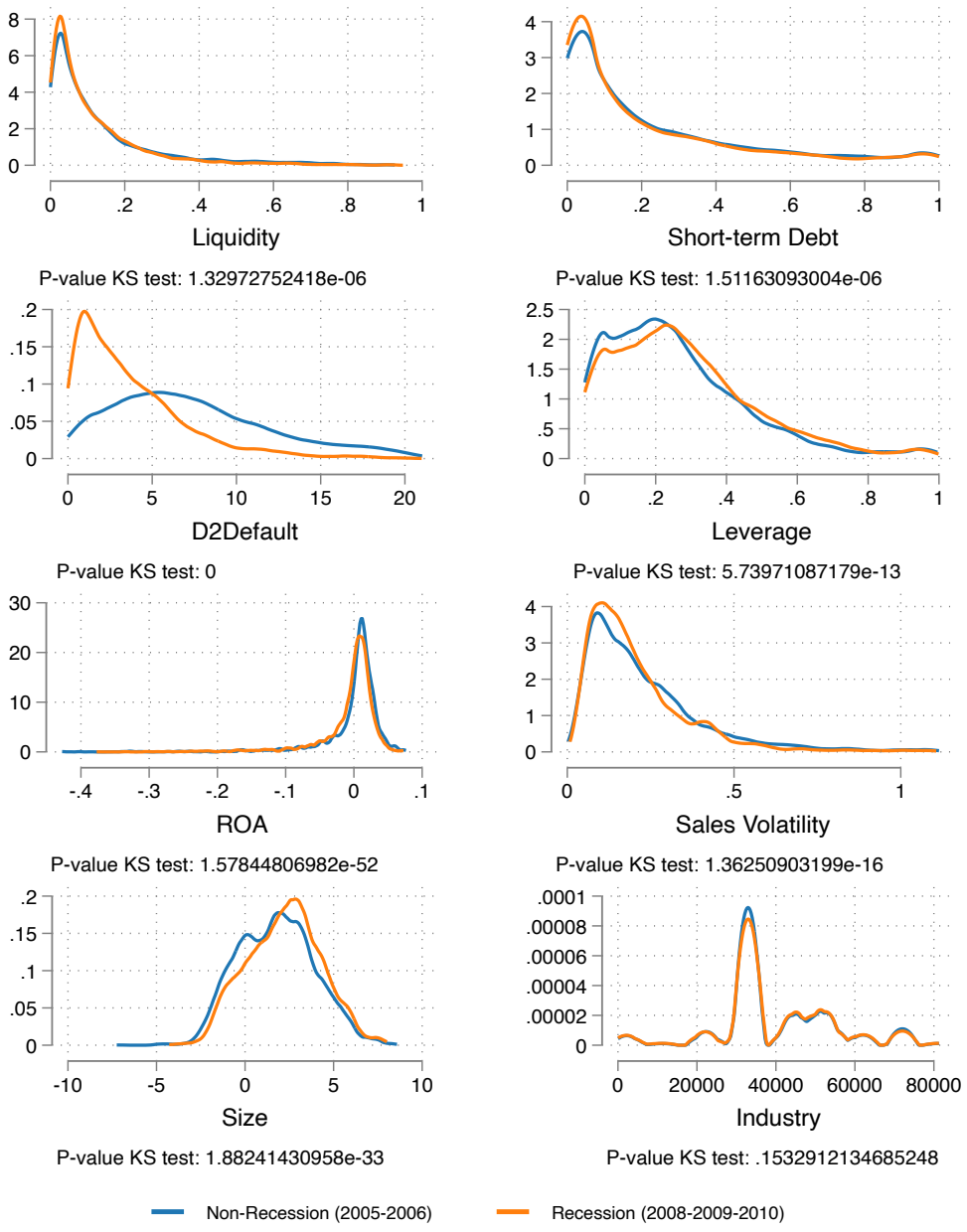
Notes: The first panel contains the summary statistics for quarterly balance sheet firm data used as independent variables. The second panel contains the summary statistics for the outcome variables. The data are from quarterly Compustat, covering 1990Q1-2018Q4. All dependent variables are trimmed at the 1.5th and 98.5th percentiles, while independent variables are trimmed at the 98.5th percentile when positive. Independent variables are linearly interpolated after cleaning steps. Units of measurement of the outcome variables are in percentage points, where 0.01 represents 1%. Additional information on variable construction can be found in Appendix A.

Table 4: Pairwise correlation matrix of balance sheet characteristics and outcome variables

Independent												
Size (log Asset)	1.000											
Leverage (Debt/Asset)	0.074	1.000										
Cash Ratio (Cash/Asset)	-0.193	-0.312	1.000									
Distance to Default	0.333	-0.373	0.142	1.000								
Short-Term Debt Ratio	-0.412	-0.218	0.130	-0.052	1.000							
Return on Asset	0.414	-0.092	-0.263	0.278	-0.210	1.000						
Sales Growth Volatility (10y)	-0.405	0.024	0.306	-0.225	0.199	-0.434	1.000					
Dependent												
Sales Growth	0.013	-0.063	0.075	0.120	-0.030	0.145	-0.005	1.000				
Market Value Growth	0.077	-0.066	0.027	0.347	-0.071	0.210	-0.074	0.256	1.000			
Investment Rate	0.038	-0.067	0.076	0.061	-0.045	0.067	-0.004	0.264	0.093	1.000		
Debt Issuance Rate	0.057	0.076	-0.061	-0.093	-0.065	-0.033	-0.008	0.147	-0.047	0.296	1.000	
Inventory Growth	0.045	-0.091	0.068	0.100	-0.042	0.135	-0.033	0.344	0.155	0.328	0.223	1.000

Notes: The first panel contains the pairwise correlation statistics for quarterly balance sheet firm data used as independent variables. The second panel contains the pairwise correlation statistics for the outcome variables. The data are from quarterly Compustat, covering 1990Q1-2018Q4. All dependent variables are trimmed at the 1.5th and 98.5th percentiles, while independent variables are trimmed at the 98.5th percentile when positive. Independent variables are linearly interpolated after cleaning steps. Additional information on variable construction can be found in Appendix A.

Figure 12: Distribution of Characteristics: Boom vs Recession



Notes: The figure illustrates the distribution of firm characteristics. The 'Non-Recession' period encompasses the years 2005-2006, while the years 2008-2009 are labeled as the 'Recession' period. The year 2007 is omitted because, according to the NBER, the recession began in December 2007, making 2007 an ambiguous year in terms of being "before" or "during" the recession. For each outcome variable-aggregate shock pair, we report the p-value of a Kolmogorov-Smirnov equality of distributions test.

B Theoretical Details

B.1 Generalized Random Forest - Algorithm

The GRF relies heavily upon the Random Forests (RF) models, since they both perform random split selection and sub-sampling. To this extent, GRF augments the methodology of RF by allowing the estimated parameters to be a weighted average of predictions, and not a pure simple average as performed in RF.

Formally, the objective of RF models is to estimate the expected value of an outcome $Y_{i,t}$, conditional on covariates $X_{i,t}$ for a given data generating process: $\mu(x) = \mathbb{E}[Y_{i,t}|X_{i,t} = x]$. The GRF aims to estimate the following moment condition:

$$\mathbb{E}[\psi_{\theta(x),\nu(x)}(O)_{i,t}|X_{i,t} = x] = 0 \quad \forall x \in \mathcal{X}, \text{and } i = 1, \ldots, n, \;\; t = 1, \ldots, T \tag{11}$$

where $O_{i,t}$ contains the set of observables, both dependent and covariates variables described in the previous section, as well as the set of exogenous shocks (W_t) that we focus on; $X_{i,t}$ represents the set of auxiliary covariates, while $\nu(x)$ is an optional nuisance parameter. Our focus is to estimate the elasticity $\hat{\theta}(x)$ for each dependent variable-shock pair, as function of all covariates.

The GRF model fits the empirical version of condition 11 by minimizing the weighted moment condition:

$$(\hat{\theta}(x), \hat{\nu}(x)) \in \;\; \text{argmin}_{\theta,\nu} \left\{ |\sum_{i=1}^{n} \alpha_i(x)\psi_{\theta,\nu}(O_{i,t})|_2 \right\} \tag{12}$$

The main additional feature of the GRF comes from the weighting function $\alpha_i(x)$: this aims to find firms with similar elasticities - depending on their characteristics $X_{i,t}$ - and associate higher weights to them. The algorithm developed by Athey et al. (2019) grows a set of B trees and defines $L_b(x)$ as the training set falling in the same "leaf" as x.

$$\alpha_{bi}(x) = \frac{\mathbf{1}(\{X_i \in L_b(x)\})}{|L_b(x)|}, \qquad \alpha_i(x) = \frac{1}{B}\sum_{b=1}^{B} \alpha_{bi}(x)$$

By bootstrapping the dataset and growing random forests, the methodology allows estimating the parameters of interest defined on many dimensions, in contrat with linear models (e.g. OLS). The interpretation of the estimated parameters $\hat{\theta}(x)$ is of a conditional local average treatement of the elasticity for a given shock.

We further estimates the average effect in the causal forests via estimates of the average partial effect, i.e. $\mathbb{E}[\text{Cov}(W_t, Y_{i,t})/\text{Var}(W_t|X_{i,t})]$. These average effects are reported in Figure 1.

B.2 Chernozhukov et al. (2018) - Test for heterogeneity

The test creates two synthetic variables, C_i and D_i:

$$C_i = \bar{\beta}(W_i - \hat{W}_i),$$
$$D_i = (\hat{\beta}^{cf} - \bar{\beta})(W_i - \hat{W}_i),$$

where the former uses only the average treatment effect while the latter is the prediction that takes into account the heterogeneity as predicted by the casual forest. The test consists in running the following regression of residuals in treatment on C_i and D_i:

$$Y_i - \hat{Y}_i = \gamma C_i + \delta D_i \tag{13}$$

The null hypothesis of the test is $\delta = 0$, which indicates that the casual forest does not capture any heterogeneity. In line with the evidence on the CV, we find that we can reject the null hypothesis of no heterogeneity in treatment effects for almost all aggregate shock-outcome variable pairs.

C Additional Figures and Tables

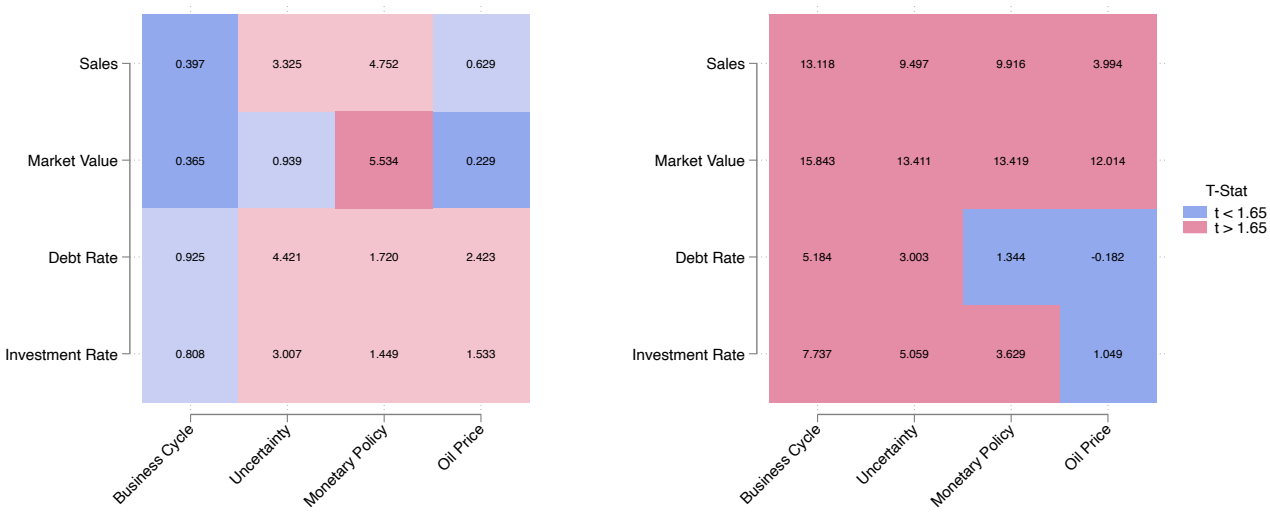
C.1 Related to Fact I

Table 5: Comparing average from linear benchmark and GRF

Dependent variable	Sales			Market Value			Debt Rate			Investment1			Investment2		
Specification	OLS	GRF	Diff %	OLS	GRF	Diff %	OLS	GRF	Diff %	OLS	GRF	Diff %	OLS	GRF	Diff %
Business Cycle	2.234	2.169	0.030	4.241	4.254	-0.003	1.208	1.270	-0.049	0.856	0.919	-0.069	1.724	1.861	-0.074
	0.066	0.855	0.076	0.124	1.612	-0.008	0.107	1.173	-0.053	0.056	0.718	-0.088	0.100	1.269	-0.108
Uncertainty	-0.228	-0.222	0.030	-1.287	-1.294	-0.006	-0.067	-0.078	-0.143	-0.072	-0.090	-0.205	-0.292	-0.343	-0.150
	0.012	0.140	-0.048	0.022	0.309	0.025	0.021	0.192	0.058	0.011	0.131	0.141	0.020	0.241	0.214
Monetary Policy	1.385	1.336	0.037	-9.769	-9.078	0.076	-0.857	-1.011	-0.153	-1.113	-0.887	0.255	-2.355	-1.905	0.236
	0.277	3.929	0.013	0.535	9.077	-0.076	0.478	4.861	0.032	0.204	2.735	-0.083	0.324	4.285	-0.105
Oil Price	-0.014	-0.015	-0.071	-0.025	-0.029	-0.122	-0.070	-0.070	-0.006	-0.041	-0.043	-0.060	-0.050	-0.052	-0.051
	0.008	0.077	0.014	0.015	0.207	0.017	0.013	0.124	0.004	0.005	0.063	0.041	0.008	0.109	0.024

Notes: The table shows the average effect of a shock on the outcome variable based on the linear benchmark model, accounting for unobserved heterogeneity at the NAICS-5 digit industry level, and the average effect based on the distribution of micro-level sensitivities to aggregate fluctuations estimated using the causal forest. The business cycle is measured as the year-to-year growth rate in real GDP per capita. Monetary policy is the monetary policy shock from Bauer and Swanson (2023) and normalized to change the 1-year government yield by 1 percent. Uncertainty is the 12-months macro uncertainty shock from Jurado et al. (2015) and normalized to change the volatility index by 1 percent in a year. Oil price is the oil price shock from Känzig (2021) and normalized to increase the oil price by 1 percent in a year. All values are deflated by the quarterly manufacturing price index.

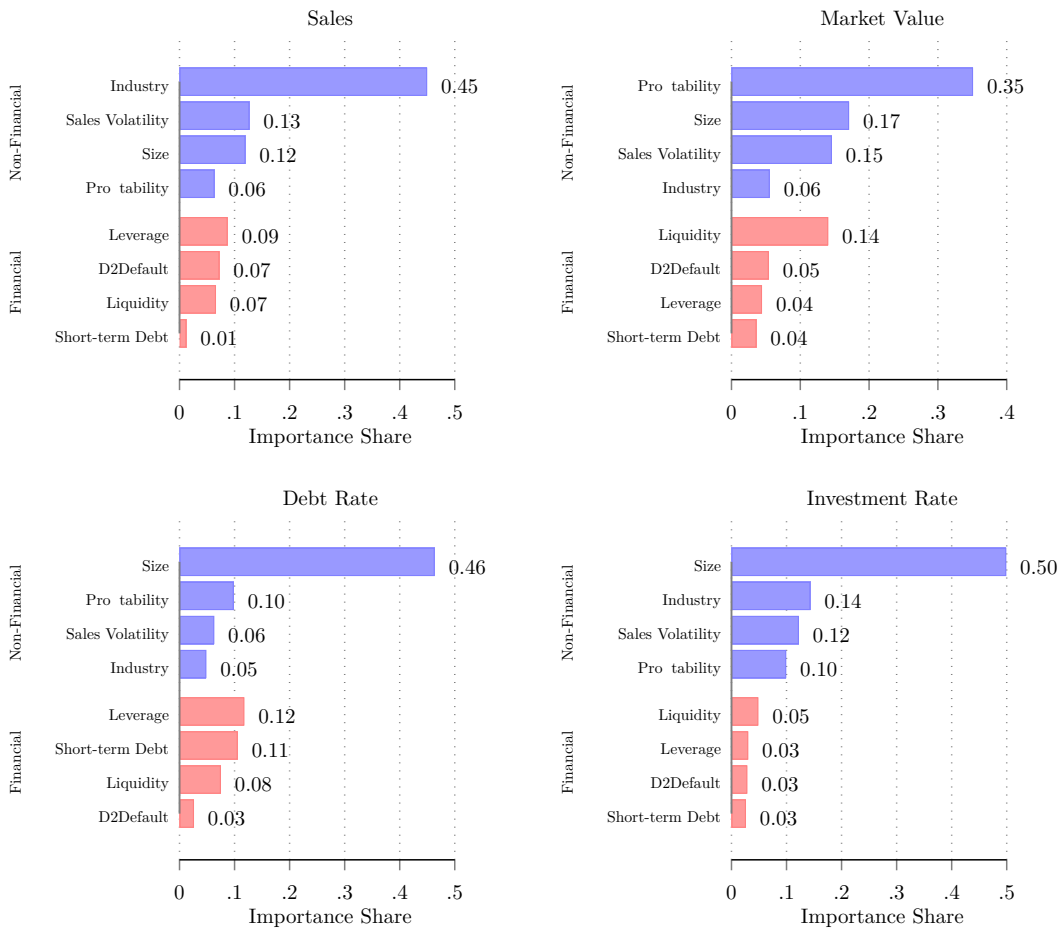
Figure 13: Heterogeneity – Coefficient of Variation and Chernozhukov et al. (2018) Test



Notes: The figure presents on the right hand side reports the median coefficient of variations of the estimated firm-level sensitivity. On the left hand side the t-statistic of the Chernozhukov et al. (2018) test for each aggregate shock - outcome variable pair. An absolute t-statistic value below 1.648 indicates no particular degree of heterogeneity, while a value above the threshold of 1.648 suggests a statistical high level of heterogeneity in firm sensitivity at a 90% confidence interval.

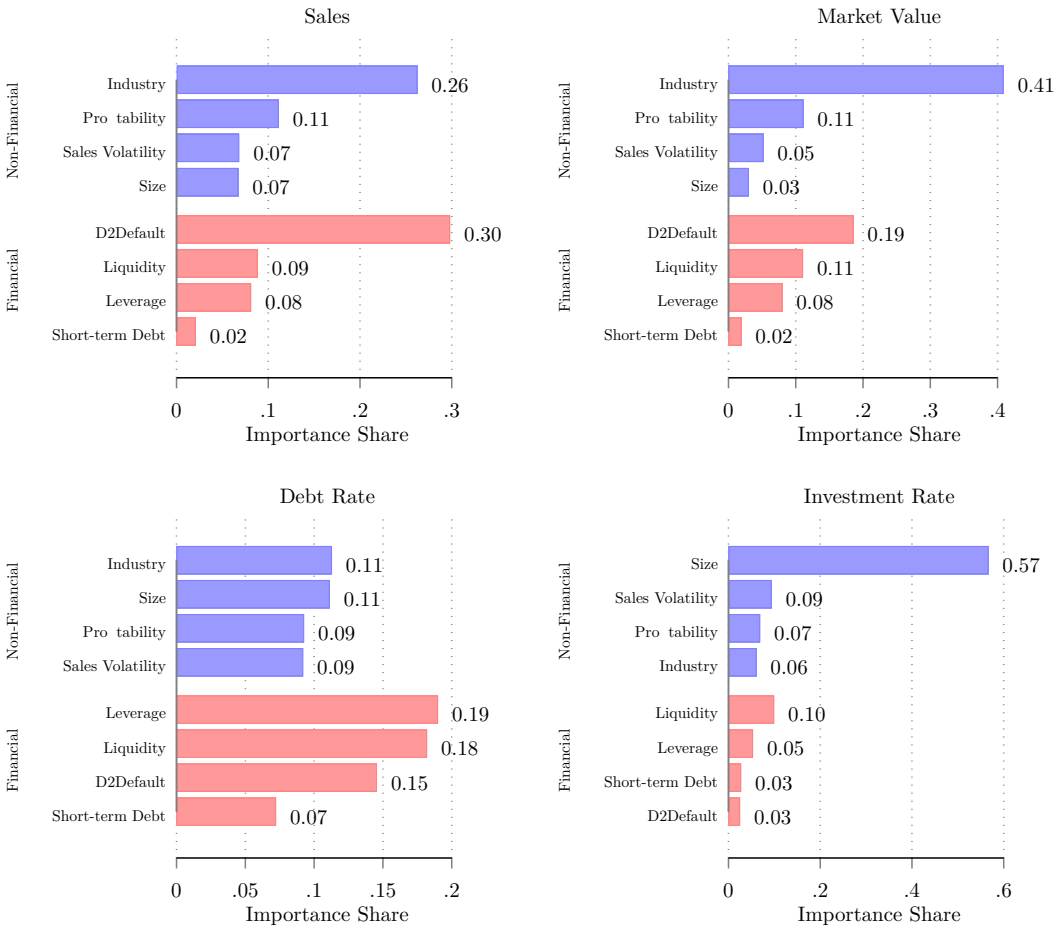
C.2 Related to Fact II

Figure 14: Importance Share for Business Cycle Fluctuations



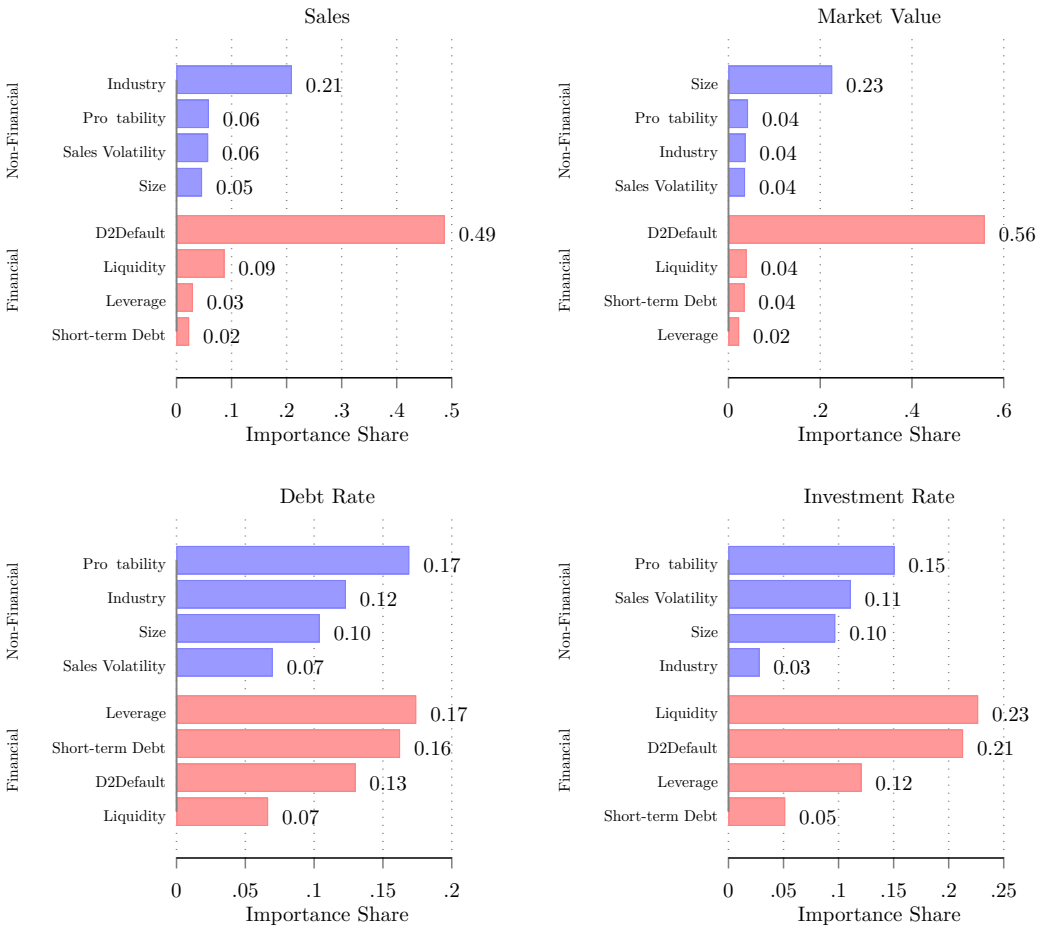
Notes: The figure displays the importance share of firms' balance sheet characteristics for all outcome variables in response to business cycle fluctuations. The importance share for the "Financial" variables is shown in red, while the importance share for the "Non-Financial" variables is in blue. Additional information on the grouping and outcome variables can be found in Appendix A.

Figure 15: Importance Share for Uncertainty



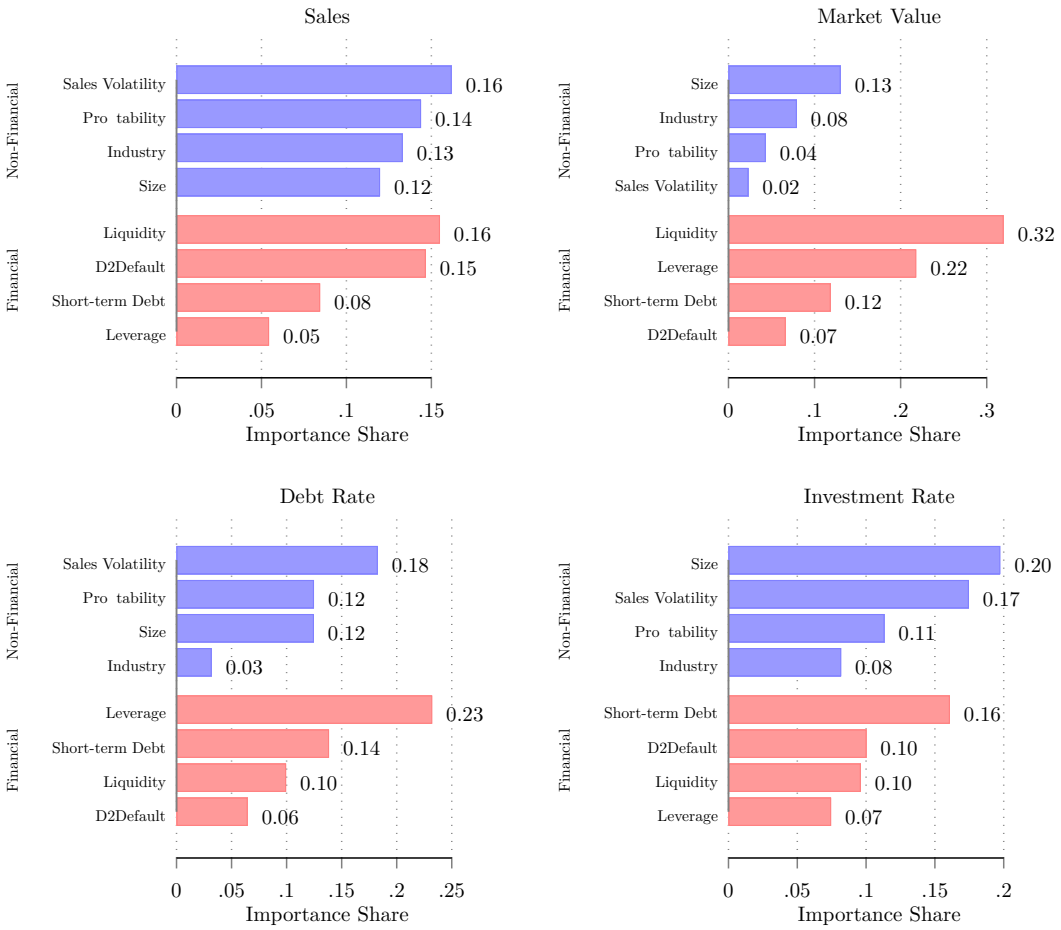
Notes: The figure displays the importance share of firms' balance sheet characteristics for all outcome variables in response to aggregate uncertainty. The importance share for the "Financial" variables is shown in red, while the importance share for the "Non-Financial" variables is in blue. Additional information on the grouping and outcome variables can be found in Appendix A.

Figure 16: Importance Share for Monetary Policy Shock



Notes: The figure displays the importance share of firms' balance sheet characteristics for all outcome variables in response to a monetary policy shock. The importance share for the "Financial" variables is shown in red, while the importance share for the 'Non-Financial' variables is in blue. Additional information on the grouping and outcome variables can be found in Appendix A.

Figure 17: Importance Share for Oil Price Shock



Notes: The figure displays the importance share of firms' balance sheet characteristics for all outcome variables in response to a oil price shock. The importance share for the 'Financial' variables is shown in red, while the importance share for the 'Non-Financial' variables is in blue. Additional information on the grouping and outcome variables can be found in Appendix A.

Table 6: Correlation Share of Importance across Dependent Variables and Shock

	(1) Sales	(2) Investment	(3) Debt Rate	(4) Market Value
Sales	1.000***	0.343***	0.223**	0.712*
	(0.000)	(0.060)	(0.113)	(0.409)
Investment		1.000***	0.292***	-0.026
		(0.000)	(0.075)	(0.317)
Debt Rate			1.000***	0.319
			(0.000)	(0.269)
Market Value				1.000***
				(0.000)

	(1) Business Cycle	(2) Monetary Policy Shock	(3) Oil Price Shock	(4) Macro Uncertainty
Business Cycle	1.000	0.103	0.103	0.442*
	(.)	(0.152)	(0.103)	(0.259)
Monetary Policy Shock		1.000***	0.054	0.175
		(0.000)	(0.089)	(0.149)
Oil Price Shock			1.000***	0.526
			(0.000)	(0.371)
Macro Uncertainty				1.000***
				(0.000)

Notes: The left panel of the table reports the regression coefficients of the following specification: $share^c_{ij} = \alpha_c + \beta share^c_{ij'} + \nu^c_{ijj'}$, where $share^c_{ij}$ is the share of importance of characteristic c in the aggregate shock i - outcome variable j pair. We consider all possible j-j' combinations, where j defines an outcome variable (sales, investment rate, debt rate, and market value). Similarly, the right panel of the table reports the regression coefficients of the following specification: $share^c_{ij} = \alpha_c + \beta share^c_{i'j} + \nu^c_{iji'}$. We consider all possible i-i' combinations, where i defines an aggregate shock (business cycle, monetary policy, macro uncertainty, and oil shock). Standard errors are clustered at firm characteristic level. The importance share is computed for each firm characteristic in each aggregate shock - outcome variable pair from the estimated casual forest.

C.3 Related to Fact III

Table 7: Additional test of non-linearity of CATEs based on AIC and R^2.

	AIC		Adjusted R^2	
	All Variables	EDF GAM > 2	All Variables	EDF GAM > 2
Sales				
All Variables	0.93	0.89	0.93	0.89
Importance Share > 10%	0.89	0.83	0.89	0.83
Market Value				
All Variables	1.00	1.00	1.00	1.00
Importance Share > 10%	1.00	1.00	1.00	1.00
Debt Rate				
All Variables	0.86	0.82	0.86	0.82
Importance Share > 10%	0.91	0.91	0.91	0.91
Investment Rate				
All Variables	0.96	0.89	0.96	0.89
Importance Share > 10%	1.00	0.93	1.00	0.93

Notes: The table reports the share of cases that reject linearity between covariates and the CATE produced by the Causal Forest based on the AIC and the adjusted R^2 obtained from a GAM model.

Table 8: Test on non-lineairity of CATEs - P-Values

	Sales	Market Value	Debt Rate	Investment Rate
Business Cycle				
Size	0.047	0.000	0.000	0.000
Leverage	0.223	0.000	0.000	0.000
Liquidity	0.000	0.501	0.000	0.000
D2Default	0.000	0.000	0.000	0.000
Short-term Debt	0.095	0.000	0.000	0.000
Profitability	0.048	0.000	0.000	0.000
Sales Volatility	0.000	0.000	0.000	0.000
Monetary Policy				
Size	0.000	0.000	0.000	0.000
Leverage	0.000	0.000	0.000	0.000
Liquidity	0.000	0.000	0.000	0.432
D2Default	0.000	0.000	0.000	0.000
Short-term Debt	0.000	0.000	0.000	0.000
Profitability	0.000	0.000	0.479	0.000
Sales Volatility	0.000	0.000	0.000	0.000
Oil Price				
Size	0.428	0.000	0.000	0.000
Leverage	0.000	0.000	0.000	0.000
Liquidity	0.000	0.000	0.000	0.000
D2Default	0.001	0.000	0.000	0.000
Short-term Debt	0.001	0.000	0.000	0.000
Profitability	0.001	0.000	0.000	0.000
Sales Volatility	0.004	0.000	0.000	0.000
Uncertainty				
Size	0.000	0.000	0.000	0.001
Leverage	0.000	0.000	0.000	0.000
Liquidity	0.000	0.000	0.000	0.009
D2Default	0.000	0.000	0.000	0.000
Short-term Debt	0.000	0.000	0.000	0.000
Profitability	0.036	0.000	0.000	0.000
Sales Volatility	0.003	0.035	0.000	0.004

Notes: The table reports the the p-values for all individual characteristic-aggregate shock-outcome variable tuples.

Table 9: Test on non-lineairity of CATEs - EDF

	Sales	Market Value	Debt Rate	Investment Rate
Business Cycle				
Size	2.692	5.602	4.942	5.754
Leverage	3.254	2.099	2.025	3.060
Liquidity	2.680	3.592	2.938	2.942
D2Default	2.028	1.681	2.405	0.000
Short-term Debt	0.000	4.929	1.852	0.708
Profitability	4.931	4.920	4.849	2.640
Sales Volatility	3.758	2.303	0.000	5.192
Monetary Policy				
Size	1.323	3.238	4.733	4.026
Leverage	1.567	2.222	7.591	3.493
Liquidity	2.939	2.126	2.220	2.951
D2Default	7.119	6.628	2.778	5.640
Short-term Debt	2.647	5.705	1.475	1.215
Profitability	3.781	5.080	1.474	4.035
Sales Volatility	2.770	2.249	1.108	4.782
Oil Price				
Size	1.515	3.440	4.771	2.971
Leverage	3.254	4.054	5.686	4.175
Liquidity	3.107	3.043	0.000	1.445
D2Default	2.931	2.412	0.000	2.444
Short-term Debt	0.982	4.980	0.000	3.074
Profitability	2.293	4.204	4.752	1.059
Sales Volatility	2.656	2.628	1.479	1.325
Uncertainty				
Size	3.157	3.613	2.867	6.308
Leverage	2.393	1.779	1.742	3.122
Liquidity	3.400	4.080	2.541	3.878
D2Default	3.018	2.814	2.793	0.011
Short-term Debt	0.000	1.604	0.546	1.953
Profitability	5.748	5.957	4.732	3.553
Sales Volatility	4.167	4.094	2.364	4.951

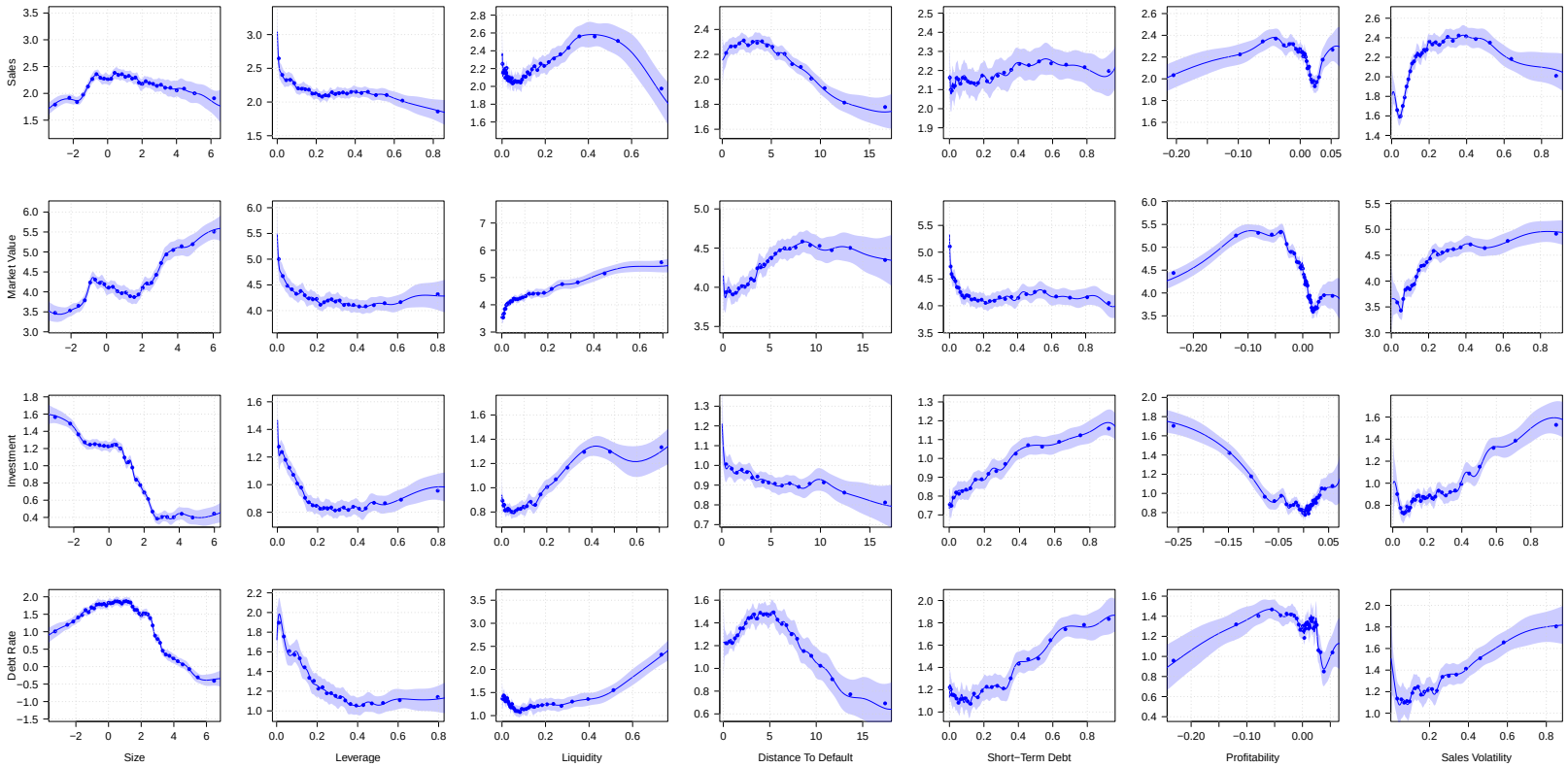
Notes: The table reports the estimated EDF for all individual GAMs estimated for each characteristic-aggregate shock-outcome variable tuple.

Table 10: Identified Treshold from Linear Regression

Fluctuations	Outcome Variable	Balance Sheet Characteristics						
		Size	Leverage	Cash	D2D	ST-Debt	ROA	Sales Vol.
Business Cycle	Sales	0.22		0.51	17.21	0.55	-0.04	0.19
	Market Value	1.79	0.15	0.03	7.44	0.07	-0.05	0.22
	Debt Rate	0.89	0.27	0.36	4.09	0.08	-0.04	
	Investment Rate	3.60	0.18	0.41		0.44	-0.19	0.96
Uncertainty	Sales	-0.24	0.55	0.44	3.64	0.55	-0.06	0.13
	Market Value	-0.72	0.44	0.07	13.95	0.25	-0.01	0.14
	Debt Rate	-0.08	0.33	0.48	4.13	0.45	-0.12	0.22
	Investment Rate	3.80	0.16	0.69	2.79	0.31	0.01	0.60
Monetary Policy	Sales	2.99	0.46	0.69	3.42	0.09	-0.01	0.21
	Market Value	-1.06	0.18	0.53	4.20	0.06	-0.04	0.27
	Debt Rate	0.82	0.66	0.70	3.95	0.38	-0.07	0.40
	Investment Rate	-0.49	0.16	0.60	3.59	0.42	0.02	0.19
Oil Price	Sales	-0.16	0.43	0.35	2.92	0.23	0.02	0.40
	Market Value	0.16	0.40	0.15	10.82	0.07	-0.19	0.81
	Debt Rate	-2.06	0.53	0.21	8.22		-0.19	0.29
	Investment Rate	0.44	0.41	0.73	4.08	0.70	-0.09	0.39

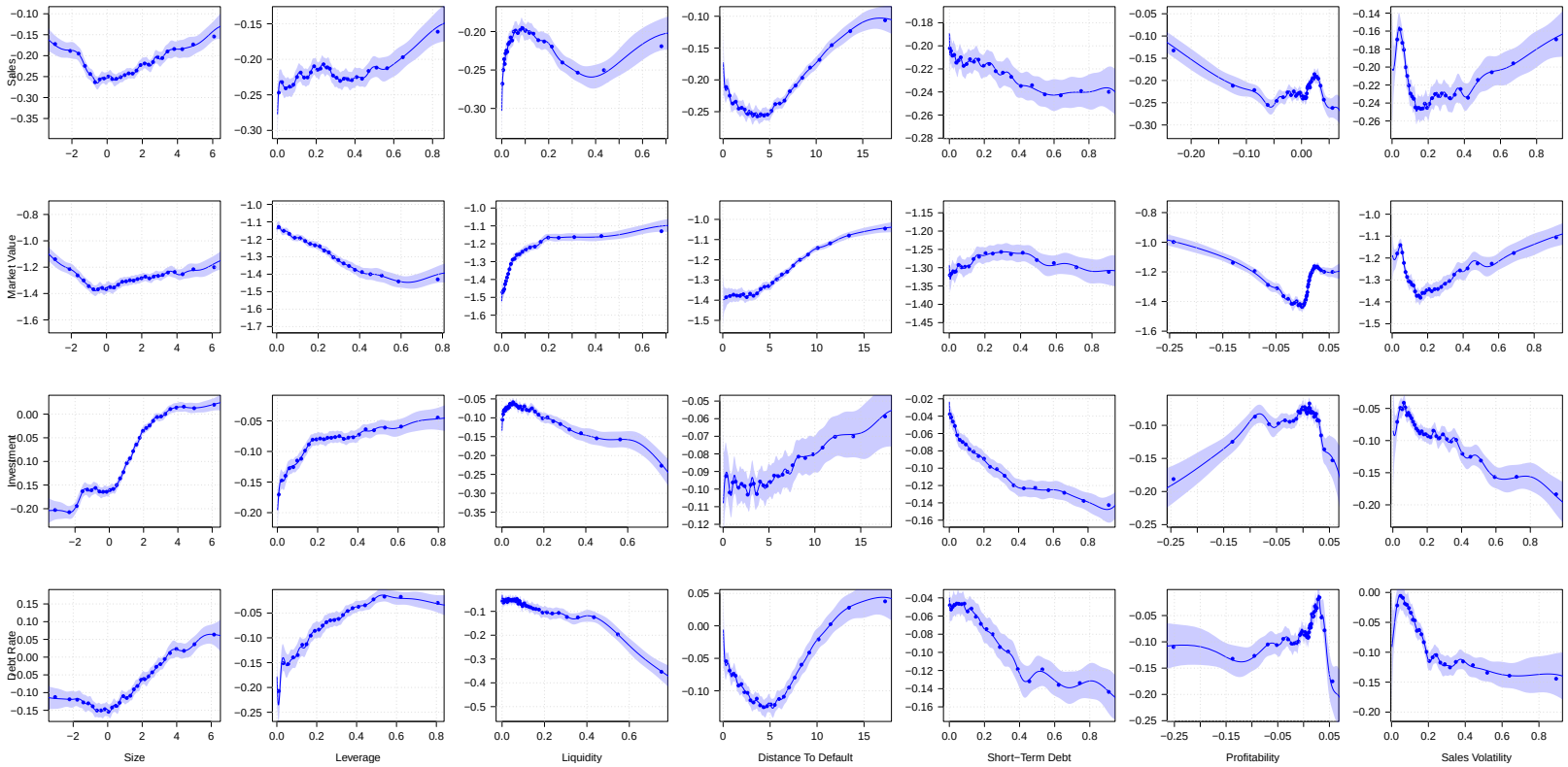
Notes: The table reports the thresholds identified in the paper. The rows show the outcome variables grouped by aggregate fluctuations, while the columns represent the balance sheet characteristics. Each cell reports the values of the thresholds identified using a segmented regression as in equation 4 in the main text.

Figure 18: Micro Sensitivities to Business Cycle Fluctuations



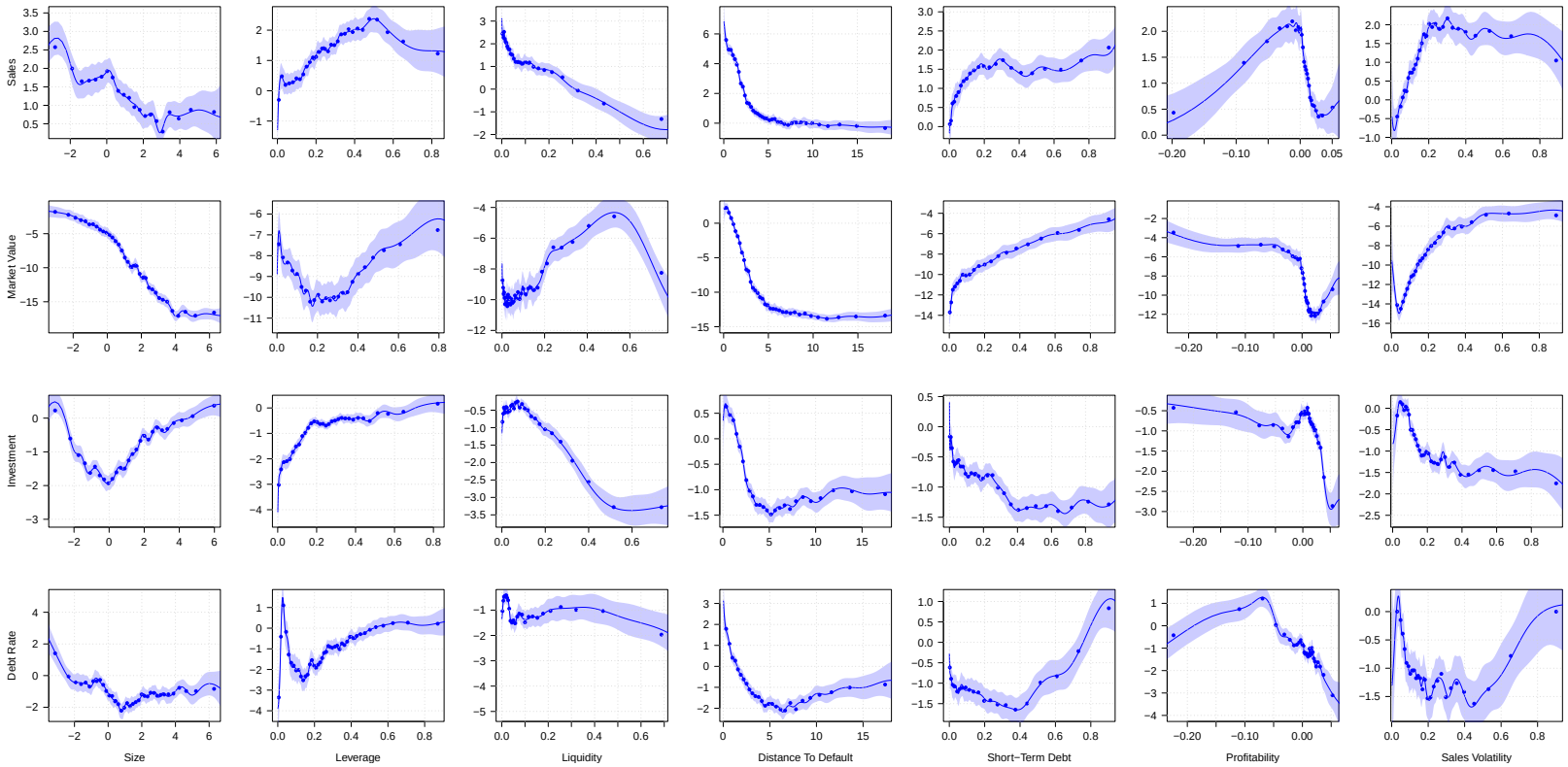
Notes: The figure illustrates the relationship between firms' sensitivity levels and their balance sheet characteristics using the regression binscatter method. The blue dots represent binned data points, while the blue fitted line depicts a cubic B-spline fit. The shaded blue areas denote confidence bands constructed using a piecewise polynomial of degree 3 with 3 smoothness constraints. The confidence bands are constructed based on 2000 simulations and 50 evenly-spaced evaluation points within each bin to approximate the supremum operator. The number of bins for partitioning the independent variable corresponds to the IMSE-optimal direct plug-in rule (Cattaneo et al., 2024). Micro sensitivities are trimmed by 0.5% on both sides and averaged at the yearly level for each gvkey before plotting. Additional information on the grouping and outcome variables can be found in Appendix A.

Figure 19: Micro Sensitivities to Uncertainty Shocks



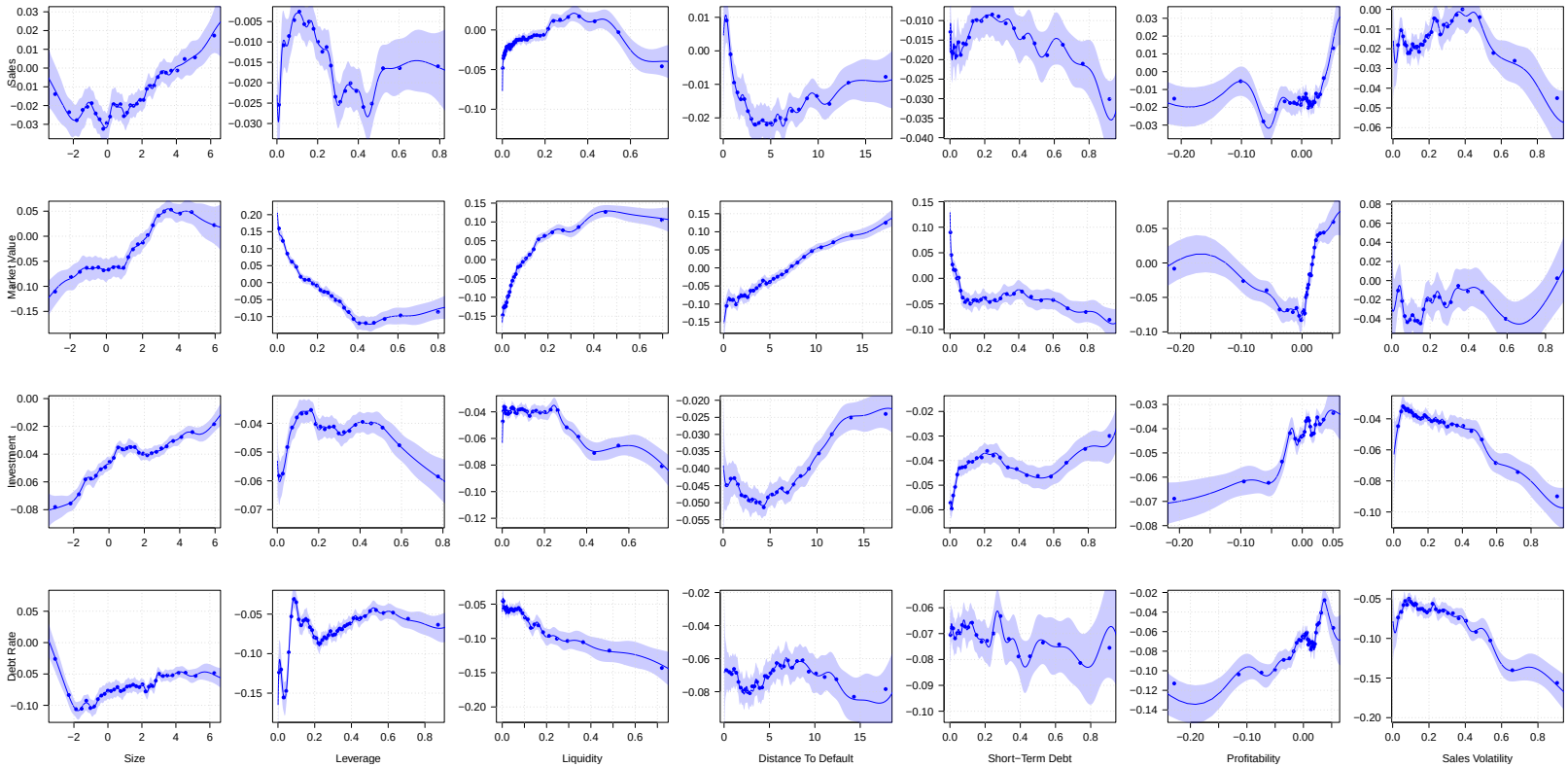
Notes: The figure illustrates the relationship between firms' sensitivity levels and their balance sheet characteristics using the regression binscatter method. The blue dots represent binned data points, while the blue fitted line depicts a cubic B-spline fit. The shaded blue areas denote confidence bands constructed using a piecewise polynomial of degree 3 with 3 smoothness constraints. The confidence bands are constructed based on 2000 simulations and 50 evenly-spaced evaluation points within each bin to approximate the supremum operator. The number of bins for partitioning the independent variable corresponds to the IMSE-optimal direct plug-in rule (Cattaneo et al., 2024). Micro sensitivities are trimmed by 0.5% on both sides and averaged at the yearly level for each gvkey before plotting. Additional information on the grouping and outcome variables can be found in Appendix A.

Figure 20: Micro Sensitivities to Monetary Policy Shocks



Notes: The figure illustrates the relationship between firms' sensitivity levels and their balance sheet characteristics using the regression binscatter method. The blue dots represent binned data points, while the blue fitted line depicts a cubic B-spline fit. The shaded blue areas denote confidence bands constructed using a piecewise polynomial of degree 3 with 3 smoothness constraints. The confidence bands are constructed based on 2000 simulations and 50 evenly-spaced evaluation points within each bin to approximate the supremum operator. The number of bins for partitioning the independent variable corresponds to the IMSE-optimal direct plug-in rule (Cattaneo et al., 2024). Micro sensitivities are trimmed by 0.5% on both sides and averaged at the yearly level for each gvkey before plotting. Additional information on the grouping and outcome variables can be found in Appendix A.

Figure 21: Micro Sensitivities to Oil Price Shocks

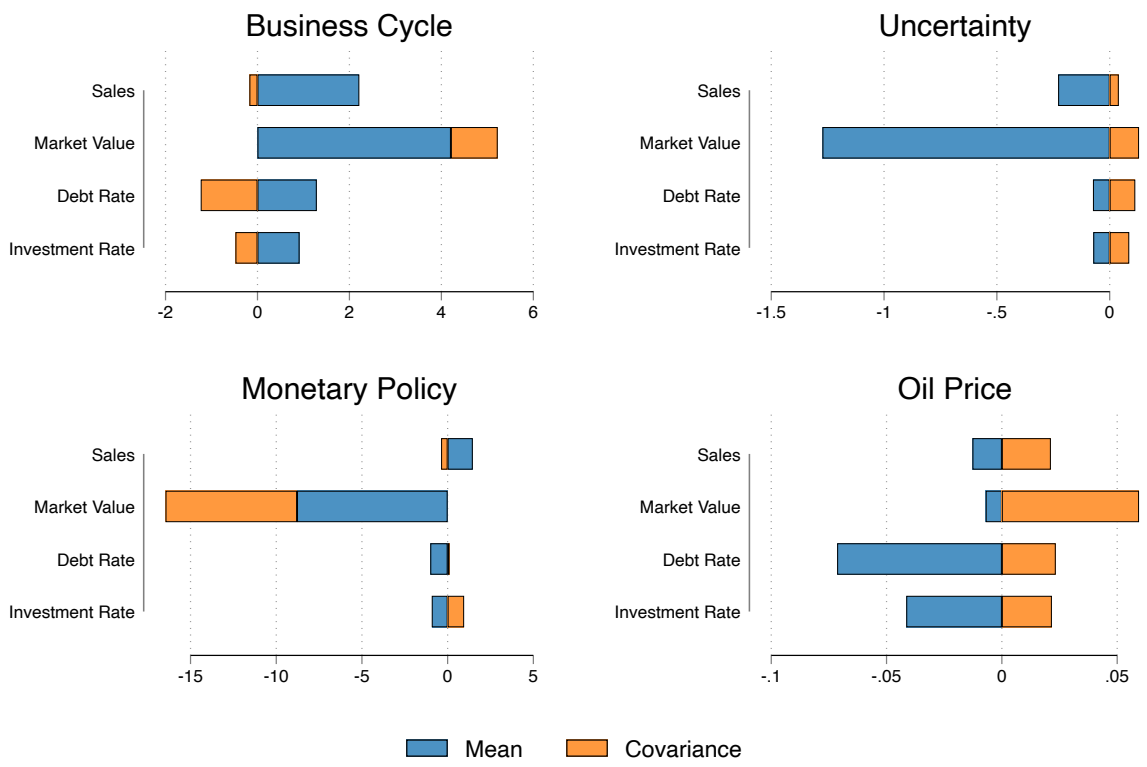


Notes: The figure illustrates the relationship between firms' sensitivity levels and their balance sheet characteristics using the regression binscatter method. The blue dots represent binned data points, while the blue fitted line depicts a cubic B-spline fit. The shaded blue areas denote confidence bands constructed using a piecewise polynomial of degree 3 with 3 smoothness constraints. The confidence bands are constructed based on 2000 simulations and 50 evenly-spaced evaluation points within each bin to approximate the supremum operator. The number of bins for partitioning the independent variable corresponds to the IMSE-optimal direct plug-in rule (Cattaneo et al., 2024). Micro sensitivities are trimmed by 0.5% on both sides and averaged at the yearly level for each gvkey before plotting. Additional information on the grouping and outcome variables can be found in Appendix A.

C.4 Additional Figures and Tables - Aggregate Results

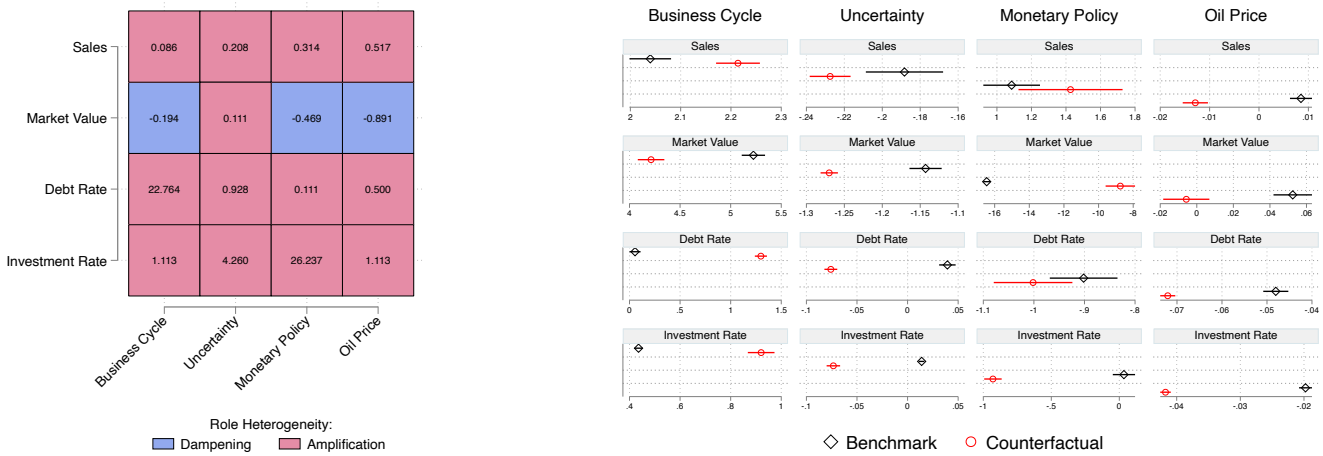
C.4.1 Heterogeneity in Sensitivities

Figure 22: Mean-Covariance Decomposition - Benchmark



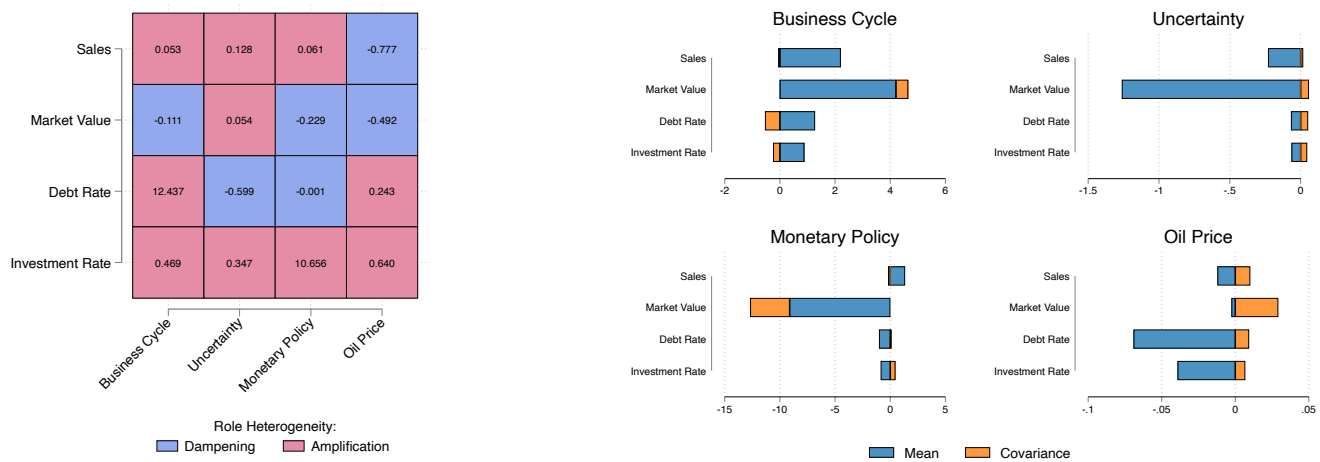
Notes: The figure plots the mean and covariance decomposition of the aggregate response for all aggregate shock-outcome variable pairs. We estimate Equation (9) using the mean and covariance terms in Equation (8) as dependent variable Z_t. The mean and covariance terms are constructed using the benchmark set of firm-level sensitivities.

Figure 23: Role of Micro-Level Sensitivity for Aggregate Response - Mean β



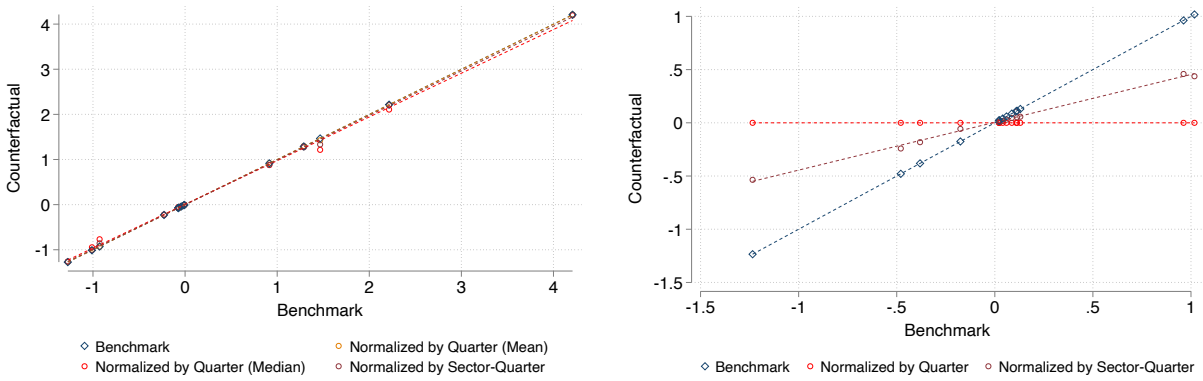
Notes: The right panel plots, for each aggregate shock - outcome variable pair, the γ coefficient estimated using Equation (9) and the benchmark aggregate response, G_t, as dependent variable (blue diamond, "Benchmark"). It also includes the γ coefficient estimated using Equation (9) and the counterfactual aggregate response, G_t^{cf}, as dependent variable (red circle, "Counterfactual"). The counterfactual aggregate response, G_t^{cf}, is constructed assuming that all firms have the same sensitivity β equal to the average value across all firms in a given quarter. The 95th percentile confidence intervals are constructed using robust standard errors. The left panel shows the ratio between the absolute value of the benchmark γ and the counterfactual γ, i.e. $\frac{|\gamma^{cf}|-|\gamma^{benchmark}|}{|\gamma^{benchmark}|}$. Red (blue) cells correspond to cases in which the aggregate response is larger (smaller) in the counterfactual scenario.

Figure 24: Role of Micro-Level Sensitivity for Aggregate Response - Median β by Sector



Notes: The left panel shows the ratio between the absolute value of the benchmark γ and the counterfactual γ, i.e. $\frac{|\gamma^{cf}|-|\gamma^{benchmark}|}{|\gamma^{benchmark}|}$. For each aggregate shock - outcome variable pair, the $\gamma^{benchmark}$ is estimated using Equation (9) and the benchmark aggregate response, G_t, as dependent variable. γ^{cf} is estimated using Equation (9) and the counterfactual aggregate response, G_t^{cf}, as dependent variable. The counterfactual aggregate response, G_t^{cf}, is constructed assuming that all firms have the same sensitivity β equal to the median value across all firms in a given industry-quarter. Red (blue) cells correspond to cases in which the aggregate response is larger (smaller) in the counterfactual scenario. The right panel plots the mean and covariance decomposition of the aggregate response for all aggregate shock-outcome variable pairs in the counterfactual scenario. We estimate Equation (9) using the mean and covariance terms in Equation (8) as dependent variable Z_t. The mean and covariance terms are constructed using the counterfactual set of firm-level sensitivities.

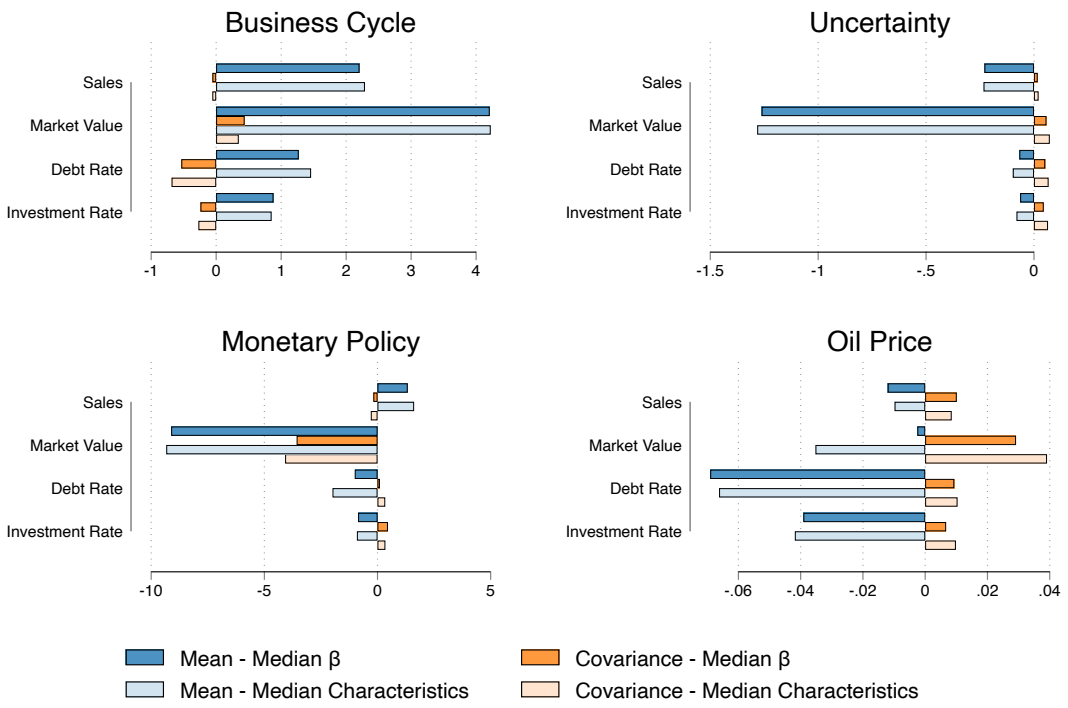
Figure 25: Comparison Mean and Covariance Coefficients - Within-Across Sector



Notes: Both panels plot the estimated counterfactual γ against the benchmark γ from specification in Equation (9). In the left (right) panel we use the mean (covariance term) from Equation (8) as dependent variable Z_t. We consider three counterfactuals: all firms have the same sensitivity β equal to the median value across all firms in a given industry-quarter ("Normalized by Sector-Quarter"); all firms have the same sensitivity β equal to the average value across all firms in a given quarter ("Normalized by Quarter (Mean)"); all firms have the same sensitivity β equal to the median value across all firms in a given quarter ("Normalized by Quarter (Median)"). Each data point represents an aggregate shock - outcome variable pair. Although the same qualitative patterns hold, we exclude the monetary policy - market value case for exposition purposes.

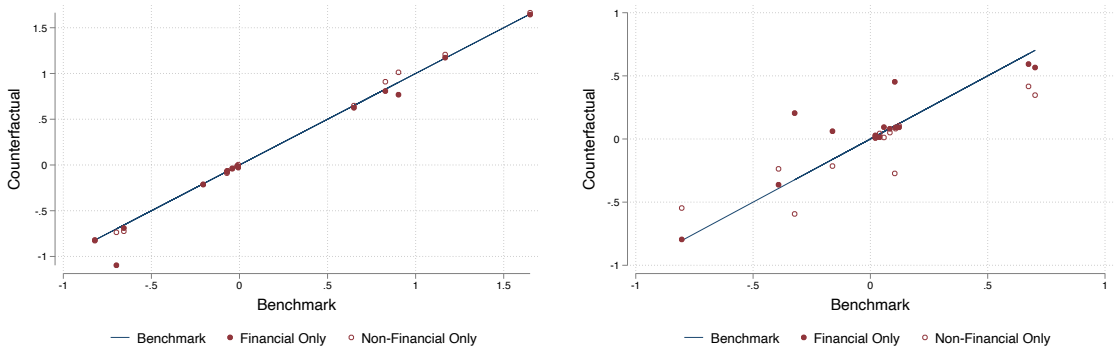
C.4.2 Heterogeneity in Characteristics

Figure 26: Role of Non-Linearities for Aggregate Response - Mean and Covariance



Notes: The right panel plots the mean and covariance decomposition of the aggregate response for all aggregate shock-outcome variable pairs in two scenarios. We estimate Equation (9) using the mean and covariance terms in Equation (8) as dependent variable Z_t. The mean (blue) and covariance (orange) terms are constructed using the counterfactual set of firm-level sensitivities. The two scenarios considered are: the counterfactual aggregate response, $G_t^{\text{median } X}$ constructed assuming that firms have the same characteristics X_{it} equal to the median in each sector-quarter (light colors, "Counterfactual - median characteristics"); the counterfactual aggregate response, $G_t^{\text{median } \beta}$, constructed assuming that firms have the same sensitivity β equal to the median in each sector-quarter (dark colors, "Counterfactual - median β").

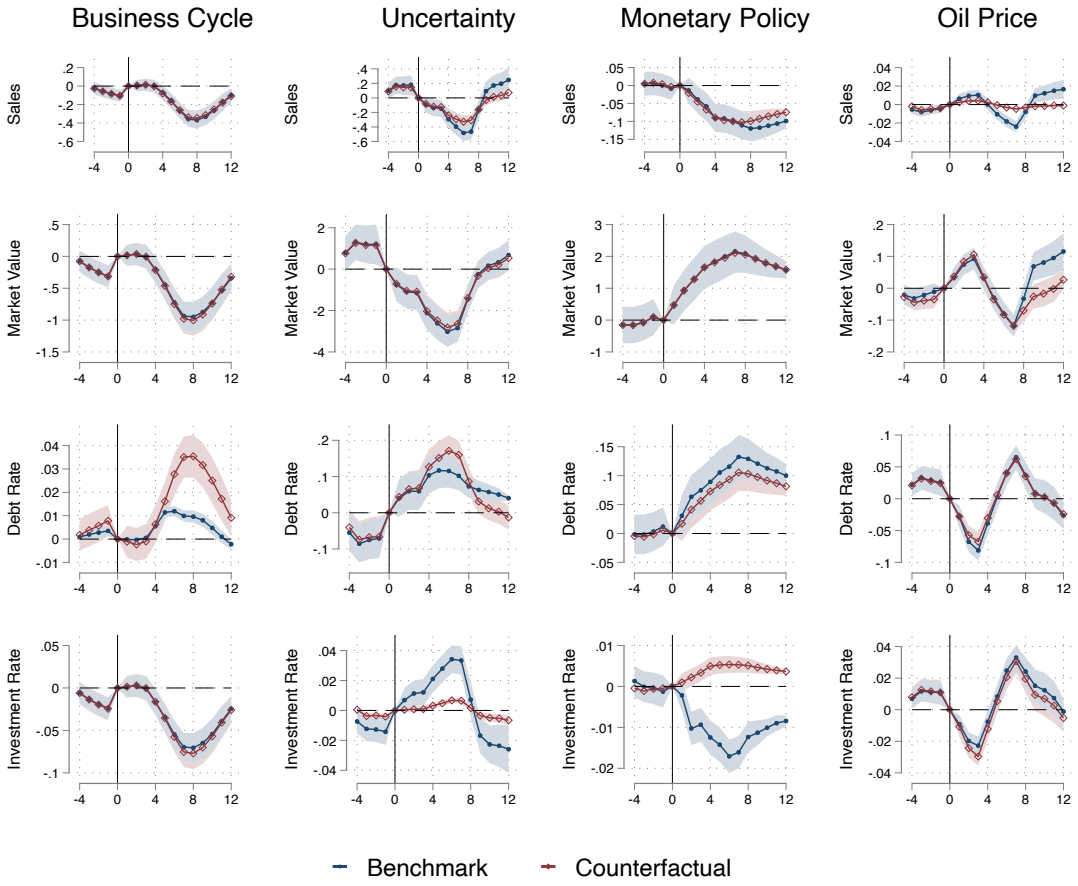
Figure 27: Role of Firms' Characteristics for Aggregate Response - Financial vs Non-Financial



Notes: Both panels plot the estimated counterfactual γ against the benchmark γ from specification in Equation (9). In the left (right) panel we use the mean (covariance term) from Equation (8) as dependent variable Z_t. We consider two counterfactuals: all firms have the same non-financial characteristics equal to the median value across all firms in a given industry-quarter (red hollow circle, "Non-Financial Only"); assuming that all firms have the same financial characteristics equal to the median value across all firms in a given industry-quarter (red full circle, "Financial Only"). Each data point represents an aggregate shock - outcome variable pair. Although the same qualitative patterns hold, we exclude the monetary policy - market value case for exposition purposes. We apply the following transformation to the ratio for exposition: $\text{sign}(x) \times \log(|x| + 1)$.

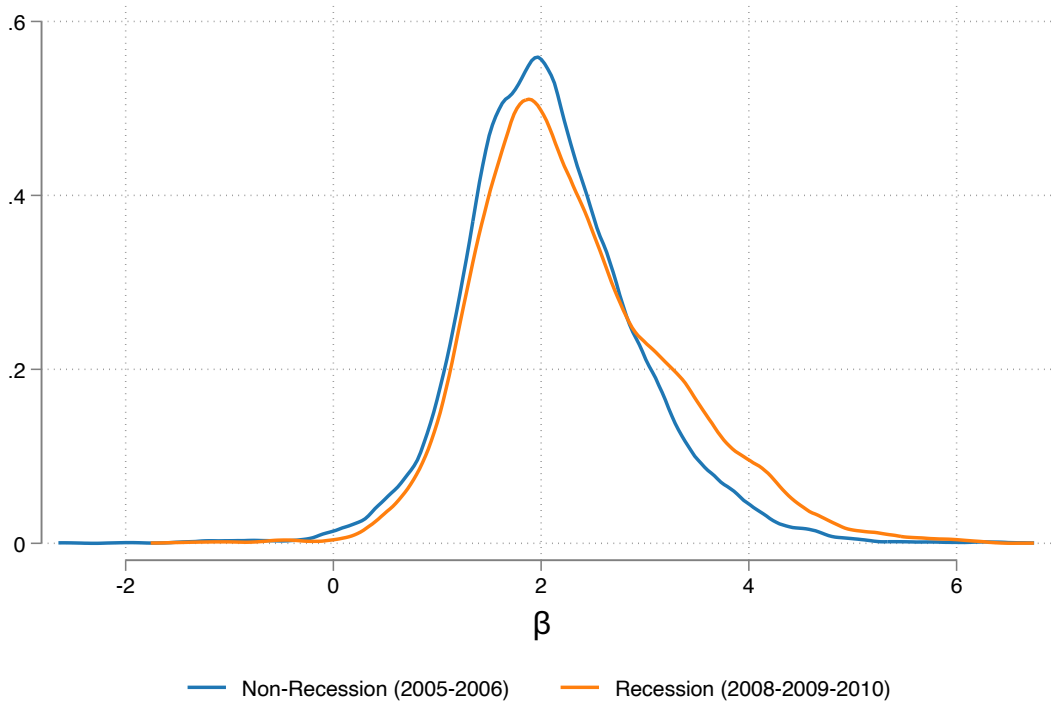
C.4.3 State-Dependence Heterogeneity

Figure 28: Aggregate Response to Aggregate Shocks in Periods of Boom vs Bust



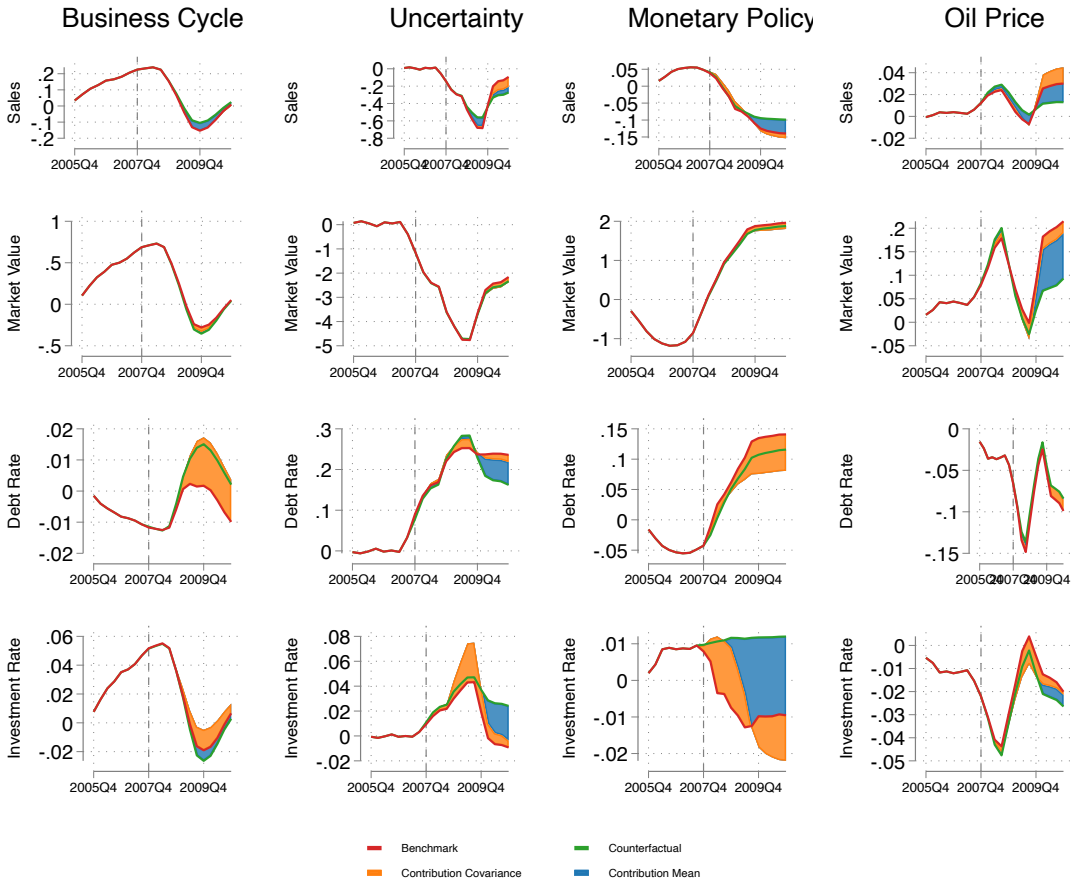
Notes: For each aggregate shock - outcome variable pair, we plot the estimated cumulative response of the outcome variable (relative to the start of the recession) using Equation (9). The benchmark aggregate response, G_t, as dependent variable (blue diamond, "Benchmark") uses the whole set of firm-level sensitivities. The counterfactual aggregate response, G_t^{cf}, as dependent variable (red circle, "Counterfactual") is constructed assuming that the all characterstics of a firm are equal to the firm average in the two years before the start of the recession. The 95th percentile confidence intervals are constructed using robust standard errors.

Figure 29: Distribution of Sensitivities Periods of Boom vs Bust



Notes: The figure illustrates the distribution of firm sensitivities. The 'Non-Recession' period encompasses the years 2005-2006, while the years 2008-2009 are labeled as the 'Recession' period. The year 2007 is omitted because, according to the NBER, the recession began in December 2007, making 2007 an ambiguous year in terms of being "before" or "during" the recession. A Kolmogorov-Smirnov equality of distributions test rejects the hypothesis of being identical.

Figure 30: Distribution of Sensitivities Periods of Boom vs Bust



Notes: For each aggregate shock - outcome variable pair, we plot the cumulative response of the outcome variable. The benchmark aggregate response, G_t, as dependent variable (red line, "Benchmark") uses the whole set of firm-level sensitivities. The counterfactual aggregate response, G_t^{cf}, as dependent variable (green line, "Counterfactual") is constructed assuming that the sensitivity of a firm is equal to the firm average in the two years before the start of the recession. The orange (blue) shaded area quantifies the contribution of the difference in the covariance (mean) terms to the aggregate difference between benchmark and counterfactual scenarios .